GAME THEORY IN EVERYDAY LIFE

From Dilemmas To Win-Win

Yung-Ming Li
Kwankamol Chaiphadung

Preface

Game Theory is the study of decision-making in situations involving interactions among multiple participants. Here, “players” are not limited to individuals — they can also be companies, organizations, or even governments and nations. Whenever choices and actions influence one another, whether in politics, economics, business, or everyday life, they can be understood as a type of game.

The appeal of game theory lies in its provision of a rigorous and systematic analytical framework, allowing us to understand how people make decisions in interactions and how these decisions shape outcomes. Especially in non-cooperative situations, where each participant acts independently and often with incomplete information, the resulting outcomes frequently diverge from our expectations. Such phenomena are commonly referred to as counterintuitive, and they often manifest as dilemmas or paradoxes: even when everyone makes seemingly rational choices, the overall result may be far from ideal.

However, the value of game theory goes beyond pointing out problems. By analyzing the structure of interactions, we can ask whether it is possible to alter the game’s framework to turn originally unfavorable outcomes into ones that are better for individuals or society. This is precisely the role game theory plays in mechanism design and strategic thinking.

Core Three-Step Approach

Learning game-theoretic thinking can be summarized in three steps:

1. Know the Game: First, understand the situation — how participants’ actions, information, and choices influence one another — and identify the core of the problem.
2. Solve the Game: Once the situation is understood, analyze potential strategies and choices, evaluating possible outcomes and equilibria.
3. Transform the Game: Finally, consider how to modify rules, incentives, or information structures to improve the equilibrium and create outcomes more favorable to individuals or society.

Structure of the Book

To guide readers step by step through the Know → Solve → Transform approach, the seasons are arranged as follows:

- **Season 0: Introduction to George Equilibrium Diaries** — Introduction to game-theoretic thinking foundation, focusing on the "**Know the Game**" step to identify players, rules, and hidden structures.
- **Season 1: Static Games** — Covers static games under complete information. This season introduces both "**Solve the Game**" and "**Transform the Game**", showing how strategic adjustments turn losing positions into Win-Win.
- **Season 2: Dynamic Games** — Explores dynamic games under complete information, using forward-looking and backward-reasoning to anticipate future moves.
- **Season 3: Static Games with Asymmetric Information** — Discusses static games where information is hidden or one side knows more than the other, while players move simultaneously.
- **Season 4: Dynamic Games with Asymmetric Information** — Combines dynamic play with asymmetric information, tackling real-world complexities like signaling and screening.
- **Season 5: Other Games** — Extends to cooperative and evolutionary games, institutional decision-making, and complex social dilemmas.

Motivation for Writing

The inspiration for this book comes from many years of teaching the course "Game Theory and Strategy" at the Graduate Institute of Information Management, National Yang Ming Chiao Tung University. In designing the course, we deliberately did not confine game theory to mathematical models or formula derivations, but instead emphasized understanding each game's situation. Only by seeing the situation clearly can one change it and affect outcomes.

In class, we often use exercises based on real-life scenarios, gradually adjusting conditions and observing how one game evolves into another. While these may appear as story changes on the surface, they actually reflect transformations in game structures and equilibria. This is the essence of mechanism design in game theory.

Years of teaching and interactions with students have shown us that game theory is not merely an analytical tool — it is a way to understand interaction structures and enhance decision-making thinking. Stories shared by past students — from life, reading, or observation — can almost always be clearly interpreted within a game-theoretic framework. It is for this reason that we compiled these experiences into a book, to share them with a broader audience.

This book, however, would not have been possible without my co-author, Kwankamol Chaiphadung. Bringing an exceptional talent for translating abstract strategic concepts into vivid, imaginative narratives, she perfectly captured the core ideas from our lectures and classroom discussions, meticulously bringing these theoretical foundations to life.

We hope this book will accompany readers as they face daily interactions and choices, helping them master the Know → Solve → Transform approach, see situations clearly, understand structures, and think about how to improve outcomes.

Yung-Ming Li
Information Economics & Business Intelligence Lab
National Yang Ming Chiao Tung University
Hsinchu, Taiwan
May, 2026

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Season 0: Introduction to George Equilibrium Diaries

In the everyday chaos of campus life, cafeteria lines, group projects, endless decisions. It is easy to feel like we are just winging it. But what if there is an invisible structure behind it all? What if every choice we make, every reaction we get, isn't random… but part of a bigger game?

Meet **George**. A first-year student who recently got admission to CCT University, where he starts a new chapter.

He is just an average guy: wide-eyed, a little lost, and happiest when holding a game controller. A gamer, not a genius. He prefers pulling combos over calculus and would much rather fight monsters than face midterms.

That all changes when he signs up for a class called "Game Theory."

At first, he thinks it is about designing video games.

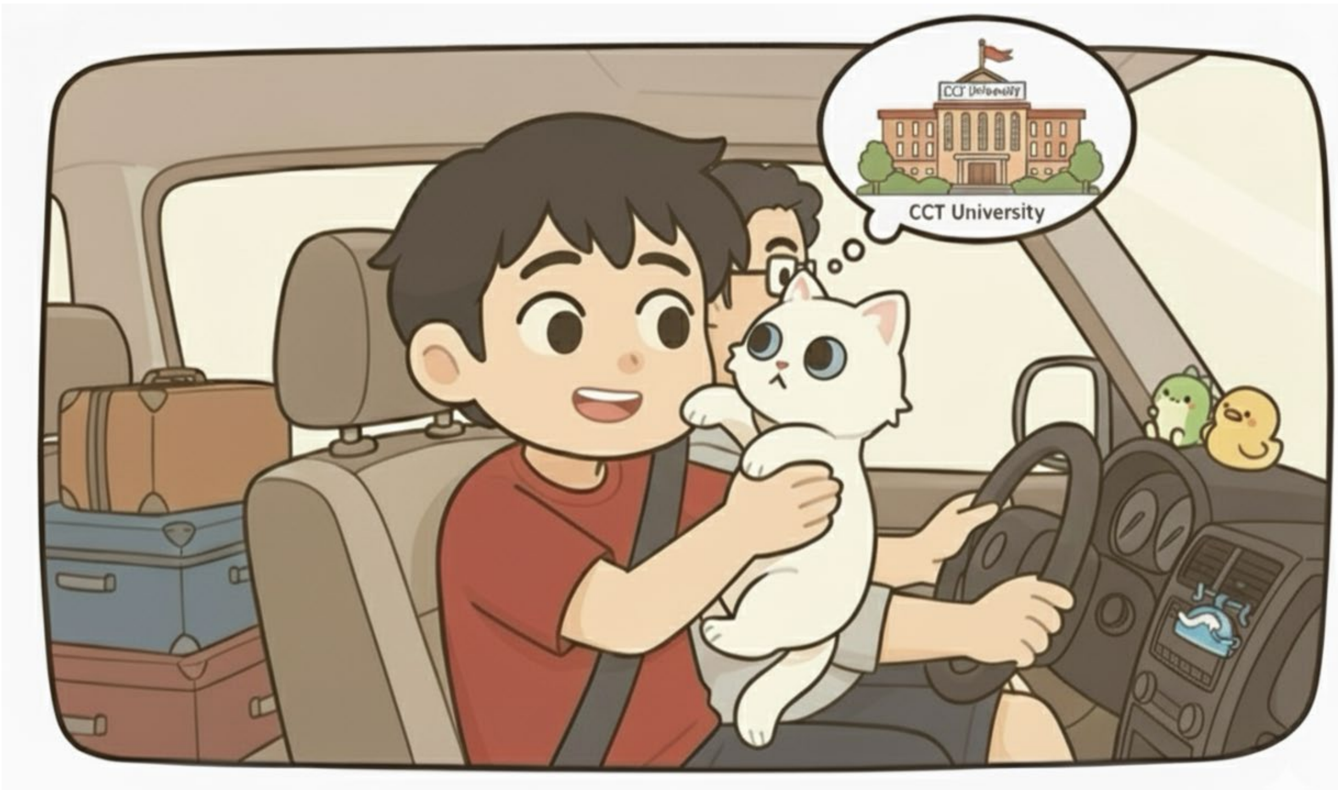
It is not.

Suddenly, he begins to see the world differently.

Whether it is battles over cafeteria seats, club elections, roommate negotiations, or awkward group projects. He realizes it is all strategy.

It is all a game. And every game has *players, strategies, and payoffs*.

Turns out, **life might be the biggest game of all**… and far more fascinating than he ever imagined.



WHAT IS A GAME?

Game Theory is a mathematical model for analyzing the decisions of individuals (or players) and the outcomes of those decisions in specific situations. In these scenarios, each player's choice and its ultimate consequences depend not just on their own action, but on the actions of all the individuals with whom that player interacts. The power of Game Theory lies in capturing this real-life tension: **what a person gets does not depend only on what they choose, but on what everyone else chooses too.**

For decades, Game Theory has been widely adopted in various fields, particularly in economics. Its applications range from:

- **Economics:** Analyzing *price wars* between two major soda companies (Coke and Pepsi) where each decides whether to cut prices to gain market share or hold them steady to maximize profit, determining optimal production levels among competing companies in an oligopoly market, and analyzing the intricate bidding behaviors in auctions.

- **Military and Political Science:** Modeling strategic conflict, such as determining effective strategies for *arms control negotiations* between two world powers, where each side decides whether to trust the other to disarm or continue building weapons.
- **Computer Science and Networking:** Designing efficient protocols for *internet traffic*, such as when thousands of users decide which server or data path to take, and the collective outcome affects the speed for everyone.
- **Law and Social Sciences:** Analyzing *settlement negotiations* in a civil lawsuit, where both the plaintiff and defendant must decide whether to negotiate a compromise or risk going to trial, which is costly for both. It is also used to study collective problems, like how neighbors decide whether to cooperate on maintaining a shared garden or free-ride on others' effort.

The Essential Components of a Game

A **game** in this sense contains three essential components:

(1) Players - Individuals or entities (such as competing firms) assumed to be **rational**. Rationality dictates that a player will always choose the action that yields them the most favorable outcome, known as their highest "payoff."

(2) Strategies - The set of available choices or actions each player can take.

(3) Payoffs - The numerical values representing the outcome or utility each player receives for every possible combination of strategies chosen by all players.

To clearly illustrate these game components, we will use the business competition matrix as an example.

> Two teenagers, A and B, want to start a small business in their village. With no prior business experience, they rely on online information to learn what might sell well. Each must decide what product to sell, knowing that their choice will affect their own profit and the other’s success. This turns a simple business idea into a strategic decision-making problem!

		Player B	
		Sell drinks	Sell food
Player A	Sell drinks	(2, 2)	(4, 6)
	Sell food	(6, 4)	(3, 3)

Figure A.1 Sell drinks or food?

In this scenario, the teenagers **A and B** act as **players**, both of whom are rational economic actors. Their strategies are their product choices: **Sell Drinks** or **Sell Food**. The outcomes are defined by **Payoffs**, the numerical utilities listed as (Payoff to A, Payoff to B). For instance, if Player A chooses to Sell Food, and Player B chooses to Sell Drinks, the payoff is **(6, 4)**. This combination yields the highest utility for A (6), as he has avoided direct competition with B (4). Conversely, if both firms sell the same thing, they both split the pool of customers and receive a divided payoff, (Food, Food) **(3, 3)** and (Drink, Drink) **(2, 2)**.

From A's perspective, if B chooses **Sell Food**, A's best response is to avoid direct competition by **Selling Drinks** as payoff of 4 is higher than 3.

		Player B	
			Sell food
Player A	Sell drinks		(**4**, 6)
	Sell food		(3, 3)

Figure A.2 Sell drinks or food: if B sells food

On the other hand, if B chooses **Sell Drinks**, A's best response is to avoid direct competition by **Selling Food** as payoff of 6 is higher than 2.

		Player B	
		Sell drinks	
Player A	Sell drinks	(2, 2)	
	Sell food	(**6**, 4)	

Figure A.3 Sell drinks or food: if B sells drinks

In conclusion, their best strategy is to stay unique by offering different types of products.

Season 1: Static Games

In these games, players act **without knowing** what the others will do. They make their decisions **simultaneously**, without prior knowledge of the others' choices.

EPISODE 1.1 - The Noise War 🔊

The Prisoner's Dilemma

As George has to move to a place close to the university, before the semester starts, he began searching for a place to live with his little white cat, Betty. He posted a short list of his requirements online… something affordable, pet-friendly, and close to campus.

Not long after, a message popped up from a guy named **Leo.**

Leo was also looking for a roommate to split the rent, and just like George… he loved cats.

The two met, checked out the apartment, and everything seemed

to click.

So, George decided to move in and share the room with Leo.

But within just three days, *before they even got a chance to grow trust*… conflict began.

George's mechanical keyboard clicks like a thousand tiny hammers.

Leo, on the other hand, tends to blast jazz and indie rock late into

the night.

"Bro, that keyboard of yours sounds like a war zone," Leo started.

"Only because your speakers are drowning me out like a nightclub," George shot back.

"Fine. I'll turn it down… if you do too."

"Deal... if you go first."

And just like that, the battle of noise began… *a quiet war of volume control.*

GAME MODELING

George & Leo's Dilemma

The newly formed roommate pair, George and Leo, immediately face a classic Game Theory conflict: the Volume Control Dilemma. Both players value a quiet apartment (mutual benefit), but both have an incentive to keep the sound loud (individual benefit).

We can model this interaction as a **simultaneous game** where the two strategies are: **Keep it Down** and **Make it Loud**. The payoffs represent their payoff for each outcome:

		Leo	
		Keep it Down	Make it Loud
George	Keep it Down	(2, 2)	(-7, 4)
	Make it Loud	(4, -7)	**(-1, -1)**

Figure 1.1.1 Keep it Down or Make it Loud?

<u>The possible outcomes:</u>

- If both **Keep it Down**, they achieve a quiet space, living happily with a moderate payoff of **(2, 2)**, the best collective outcome.
- If one **Makes it Loud** while the other cleans, the loud person enjoys full comfort (payoff of 4), while the quiet person suffers the worst penalty (payoff of -7).
- If both **Make it Loud**, the result is mutual chaos and a tense atmosphere, leading to the sub-optimal payoff of **(-1, -1)**.

The Search for a Dominant Strategy:

To predict the outcome, we analyze the choices from each player's rational perspective.

George's perspective:

- If Leo chooses **Keep it Down**, George must compare 2 (Keep it Down) versus 4 (Make it Loud). George's best response is **Make it Loud** (4 > 2).
- If Leo chooses **Make it Loud**, George must compare -7 (Keep it Down) versus -1 (Make it Loud). George's best response is still **Make it Loud** (-1 > -7).

Since **Make it Loud** yields a higher payoff for George regardless of Leo's action, **make it Loud** is the **Dominant Strategy** for George.

Leo's perspective:

- If George chooses **Keep it Down**, Leo must compare 2 (Keep it Down) versus 4 (Make it Loud). Leo's best response is **Make it Loud** (4 > 2).
- If George chooses **Make it Loud**, Leo must compare -7 (Keep it Down) versus -1 (Make it Loud). Leo's best response is still **Make it Loud** (-1 > -7).

Similarly, **make it Loud** is the **Dominant Strategy** for Leo.

Nash Equilibrium

Since both players have a dominant strategy: **Make it Loud**, the expected and final result of the game is the combination **(Loud, Loud)**, resulting in the payoff of **(-1, -1)**.

This point is the **Nash Equilibrium** of the game. It predicts that both George and Leo will suffer from mutual chaos, even though they both know they would be significantly better off if they had cooperated and chosen **Keep it Down** (2, 2). This structural failure is the fundamental insight of the Prisoner's Dilemma.

Dominant Strategy

A key concept here is the strictly **Dominant Strategy**. This is a strategy that gives a player a strictly better payoff than any other strategy, *regardless* of what the other players choose. We also can say that dominant strategy is **the best response** to every strategy of other players. In this case, "Make it Loud" is the strictly dominant strategy for both George and Leo. If a player has a strictly dominant strategy, a rational player will always choose it.

Nash Equilibrium

A **Nash Equilibrium** is a set of strategies where no player can improve their outcome by unilaterally changing their strategy. It is the primary concept to solve any game scenario.

Pure Strategy

A **Pure Strategy** means each player chooses one specific action, no randomness. So, the outcome where both choose to (Loud, Loud) (-1, -1) is the **Pure Strategy Nash Equilibrium** for this game.

Before Nash Equilibrium Concept

Although we established that the **Nash Equilibrium** is the primary concept, the original way to solve strategic games was the **Dominant Strategy**. This method used to be the default solution for any game scenario at that time.

The Dominant Strategy can uniquely predict the outcome of a game by **eliminating the strictly dominated strategy**, which the one that will always be replaced by a superior strategy because the superior one performs better regardless of what the opponent does.

For example, George and Leo are playing a simulating game, where they are trapped in an old building during a blackout. There are **two emergency routes**, but they must choose **at the same second**, without communicating. **George** controls a lever that moves the platform **Up** or **Down**, while Leo controls a switch that sends the platform **Left** or **Right**.

		Leo	
		Left	~~Right~~
George	Up	(**5**, *3*)	~~(**1**, *1*)~~
	~~Down~~	~~(**2**, *2*)~~	~~(**0**, *-1*)~~

Figure 1.1.2 Up or Down/Left or Right?

- For **George**, playing Up always gives him a better payoff: **5 and 1**, compared to Down: **2 and 0**.
- For **Leo**, playing Left always gives him a better payoff: *3 and 2*, compared to Right: *1 and -1*.

The Dominant Strategy concept predicts that George will play **Up**, Leo will play **Left**.

		Leo
		Left
George	Up	(**5**, *3*)

Figure 1.1.3 George plays Up, Leo plays Left

In the method of Iterated Elimination of Dominated Strategies, the process is simple: continue reducing the inferior strategies until only one is left. This final outcome is the dominant strategy for each player and the predicted solution for the game. This method can be used for a large number of players and strategies.

However, the issue arose because many special scenarios **do not have a Dominant Strategy**, meaning there was no solution. For example, if George's best response depends entirely on what Leo chooses, and vice-versa, there is no strategy that is always superior.

The concept of **Nash Equilibrium** emerged to solve this issue, providing a powerful solution for complex problems where players must consider their opponents' best response. This concept can be used to solve almost all of the scenarios in game theory, including special scenarios that do not have a Dominant Strategy.

Despite its power, game theory does not always have to use the Nash Equilibrium concept. There are some special, interesting games that cannot be solved effectively using standard competitive equilibrium analysis, which we will mention in Season 5: Other Games.

Pareto Optimality and Social Optimality

While the Nash Equilibrium shows us where rational individuals will land, it does not always guarantee the *best* outcome for the group. This leads us to consider two important concepts for evaluating outcomes:

- **Pareto Optimality:** An outcome is Pareto-optimal if no player can be made better off without making someone else worse off. In a Pareto optimal state, all "easy" improvements have been exhausted.

		Leo	
		Keep it Down	Make it Loud
George	Keep it Down	***(2, 2)***	(-7, 4)
	Make it Loud	(4, -7)	**(-1, -1)**

Figure 1.1.4 Finding Pareto and Social Optimality

In the George and Leo's Dilemma, the outcome where both **(Down, Down)** ***(2, 2)*** **is Pareto optimal**. To make either George or Leo better off than 2 (e.g. getting 4), the other would have to get –7, making them *worse* off. In contrast, the Nash Equilibrium outcome of **(Loud, Loud) (-1, -1)** is *not* Pareto optimal, because we *can* make both George and Leo better off (moving to (*2, 2*)) without hurting anyone. Thus, we would say that **Pareto Optimality is the fair, Win-Win outcome for most scenarios.**

- **Social Optimality:** An outcome is socially optimal if it maximizes the *sum* of all players' payoffs (e.g. minimizes total punishment.) In the George and Leo's Dilemma, if we sum the payoffs for each outcome:
    - **(Down, Down): 2+2 = 4**
    - (Down, Loud): -7+4 = −3
    - (Loud, Down): 4+(-7) = −3
    - (Loud, Loud): −1+(−1) = −2

    The outcome **(Down, Down)** *(2,2)*, with a sum of 4, provides the highest total payoff. Therefore, it **is the socially optimal outcome**. In this specific case, the outcome is both Pareto and socially optimal — a PERFECT outcome.

The Relationship Between the Two Optimality

While every socially optimal outcome is Pareto optimal, **not every Pareto optimal result is socially optimal**. Consider a modified matrix where the payoffs for "one-sided loudness" are greatly increased:

		Leo	
		Keep it Down	Make it Loud
George	Keep it Down	***(2, 2)***	(-7, 15)
	Make it Loud	(15, -7)	**(-1, -1)**

Figure 1.1.5 Increasing payoffs for “one-sided loudness” from 4 to 15

In this version:

- **Social Optimality:** The outcomes (Down, Loud) and (Loud, Down) are now the socially optimal results, as their total payoff is **8** (15-7).
- **Pareto Optimality:** (Down, Down) ***(2, 2)*** is **still Pareto optimal** because you cannot increase someone's payoff to 15 without decreasing the other person's payoff from 2 to -7.

This reveals a deeper strategic tension: **a result can be "efficient" (Pareto) without being the "best for the total group" (Social)**, or vice versa. To reach the Pareto optimal outcome of (2, 2) in this version, players might prefer it for its **fairness**, even though it does not maximize the total sum of the system.

Prisoner's Dilemma

The Prisoner's Dilemma is a foundational game in game theory. The actual matrix is:

		Player B	
		Stay Silent	Confess
Player A	Stay Silent	***(-1, -1)***	(-10, 0)
	Confess	(0, -10)	**(-4, -4)**

Figure 1.1.6 Prisoner's Dilemma

Player A and B are both suspects in a robbery. They are held in *separate rooms*. Each is offered the same deal, stay silent or confess.

<u>The possible outcomes:</u>

- If both stay silent, they can only be charged with resisting arrest. Both get 1 year in prison (–1, –1).
- If one confesses and the other stays silent, the confessor goes free (0), while the silent one takes full blame (–10).

- If both confess, they both receive a reduced sentence for cooperation, so both get 4 years in prison (–4, –4).

Thus, the Nash Equilibrium is **(Confess, Confess) (–4, –4)**.

George and Leo's dilemma reflect this classic Prisoner's Dilemma. It includes:

(1) Players: George and Leo *(Player A and B)*

(2) Strategies: Keep it Down or Make it Loud *(Stay Silent or Confess)*

(3) Payoffs: Level of personal satisfaction or disturbance *(Years in prison)*

Just like in prison scenarios, each player wants what is best for themselves.

		Leo	
		Keep it Down	Make it Loud
George	Keep it Down	***(2, 2)***	(-7, 4)
	Make it Loud	(4, -7)	**(-1, -1)**

Figure 1.1.7 Why it is called a Dilemma?

However, even though **(Down, Down)** ***(2, 2)*** is clearly better for both, this outcome **is not stable**. Since this is a Static Game setting, players make decisions **simultaneously**, without communication, and without knowing the other's choice in advance. Each player is separated and independent from each other.

- If George believes Leo will keep it down too (landing at (Down, Down) (2, 2)), George has an incentive to change to Loud (4 > 2) because it gives him a higher payoff. That move would shift them to (Loud, Down) (4, -7) from George's perspective.
- Leo has the same incentive. So, both abandon keeping noise down and end up at **(Loud, Loud)** (-1, -1). Not because it is the best, but because it is the safest individually.

That is why it is called a "*Dilemma*": fear of betrayal leads them to make a decision that is worse for each, creating a stable point at (–1, –1), which is neither pareto nor socially optimal. **(Down, Down)** ***(2, 2)***

— a perfect outcome that is both Pareto and socially optimal — **is impossible to achieve without trust or changing the game itself**.

GAME TRANSFORMATION

How to Solve the Dilemma and Reach the Win-Win outcome?

There are two methods to solve the prisoner's dilemma, and achieve a "Win-Win" result — a **Pareto optimal outcome** and, only in this case, socially optimal outcome. These strategic solutions are universal and can be applied to any game discussed in this book. The core approaches are simple: establishing cooperation to **find a mutually beneficial path, either by building trust or by changing the ingredients** of the game to support collective success.

1. Establish Cooperation – Letting players build trust through communication or a binding agreement can break the cycle of conflict: George and Leo might agree to keep the volume low during specific study or work hours. Once they cooperate, the matrix disappears. There is no more game, just peace.

2. Change the Environment – This would entirely turn the game into a new version of it:

One lucky day, Leo gets a brand-new Bluetooth headset. Noise-cancelling. Sleek. He is thrilled to use it everywhere… even while sleeping. Now Leo can blast his favorite tunes without bothering George, and he cannot even hear George's mechanical keyboard anymore.

		Leo	
		Keep it Down	Make it Loud
George	Keep it Down	***(2, 2)***	(-7, 4)
	Make it Loud	(4, -7)	**(4, 4)**

Figure 1.1.8 A new version of the Noise War

Dilemma solved. They can both enjoy what they love **without conflict**. A “Win-Win” by design. **The equilibrium of (Loud, Loud) (4, 4) is now Pareto** — also a socially optimal outcome *by chance*. No player has incentive to deviate alone.

In conclusion, when the conditions are right, the outcome changes. Not every game creates dilemma. However, while Win-Win represents fairness, **it does not always mean equal**. Let’s imagine a twist: What if *only* Leo had the fancy headphones, but they were *not noise-cancelling*?

> When Leo changes music…even just for a moment, he still hears the click-clack of George’s keyboard. It is bearable… but it can lower Leo’s happiness. Meanwhile, George enjoys his games day and night, totally undisturbed.

		Leo	
		Keep it Down	Make it Loud
George	Keep it Down	***(2, 2)***	(-7, 4)
	Make it Loud	(4, -7)	**(4, 1)**

Figure 1.1.9 A non-equal equilibrium

The equilibrium is (Loud, Loud). George is very happy (**4**), Leo is okay (**1**) but not great. This is still a Win-Win, fair outcome, *but not really equal*. This is where Game Theory reveals something powerful: **Sometimes, fairness means someone still gets less than the other.**

Application

Arms Race

Two rival countries must decide whether to Arm (build up military power) or Disarm (reduce military spending). The best, cooperative outcome is both Disarm (5, 5). They save a huge amount of money, and security is high. If one Arms, while the other Disarms. The Arming country achieves maximum superiority and security (8), while the Disarming country is vulnerable (-10). The fear of being harmed and the temptation for the power push them to Nash Equilibrium of (0, 0): Both Arm.

		Country B	
		Disarm	Arm
Country A	Disarm	***(5, 5)***	(-10, 8)
	Arm	(8, -10)	**(0, 0)**

Figure 1.1.10 Disarm or Arm?

To break the dilemma, they may cooperate by signing Arms Treaties (e.g. START) to build mutual trust and using third-party inspectors to ensure compliance. We may also change the game environment by utilizing UN Sanctions or a strong military alliance (like NATO) to punish any country that attempts to arm secretly. This raises the cost of Arming so high that cooperation becomes the new rational choice.

Price Competition

		Pepsi	
		High Price	Low Price
Coke	High Price	***(6, 6)***	(-20, 10)
	Low Price	(10, -20)	**(0.5, 0.5)**

Figure 1.1.11 High Price or Low Price?

This scenario models an **oligopoly**, a market dominated by a few large firms like Coke and Pepsi. They must decide whether to charge a **High Price** (Cooperate, maximizing industry profits) or charge a **Low Price** (Less profits, attempting to capture market share). The best outcome is they charge a High Price (6, 6), great profits. Crucially, while this is the optimal outcome for the firms, the negative impact on consumer is excluded from the matrix, as the model focuses exclusively on the incentives and payoffs of the players, Coke and Pepsi.

If one firm lowers its price, it steals massive market share and profit (10), severely punishing the high-price firm (-20). Nash Equilibrium is both charging a Low Price (0.5, 0.5), resulting in a price war that reduces profit margins for both, but it benefits customers.

Publicly signal pricing intentions to let another player know your future move would solve the dilemma of (Low Price, Low Price). Moreover, companies might invest in **Product Differentiation**: branding, unique features. This changes the game's condition, as customers stop choosing based solely on price.

EPISODE 1.2 - Netflix or Disney+ 📺 ?

The Coordination Game

A week has passed since George and Leo solved their noise problem. Things are finally chill…

Until one night.

George plops onto the couch with a bowl of popcorn.

“Hey,” he says, “you want to get a family plan for streaming? We can split the cost.”

Leo nods. “Totally. But... Netflix or Disney+?”

George grins. “I love Netflix. But I mean… Disney+ has Marvel and Pixar.”

Leo scratches his head. “Netflix has Stranger Things though.”

They both pause.

“So… what do we do?”

However, there is one thing *they didn’t tell each other*.

They both actually prefer Netflix.


Netflix or Disney+?
N
N

GAME MODELING

Unbalanced Coordination Game

After struggling with the Volume Control Dilemma, George and Leo face a new challenge: sharing streaming subscriptions to save money. They can only afford one plan. If they **both pick the same one**, they save money and can watch together. But if they choose **different ones**, no shared account, and double cost. Oops.

This is a **Coordination Game**, where the primary goal is to match the other player's strategy. Their payoffs are structured as follows:

		Leo	
		Netflix	Disney+
George	Netflix	**(3, 3)**	(0, 0)
	Disney+	(0, 0)	**(1, 1)**

Figure 1.2.1 Netflix or Disney+?

The possible outcomes:

- If both choose **Netflix**, they achieve the highest mutual payoff **(3, 3)**, saving money and enjoying their top-preference shows together.
- If both choose **Disney+**, they still successfully coordinate, resulting in a moderate, successful payoff of **(1, 1)**.
- If they choose differently (one Netflix, one Disney+), they fail to share the plan. This forces them to pay full price individually, resulting in the worst payoff of **(0, 0)**.

<u>The Search for a Dominant Strategy:</u>

To predict the outcome, we analyze the choices from each player's rational perspective.

George's perspective:

- If Leo chooses **Netflix**, George must compare 3 (Netflix) versus 0 (Disney+). George's best response is **Netflix** (3 > 0).
- If Leo chooses **Disney+**, George must compare 0 (Netflix) versus 1 (Disney+). George's best response is **Disney+** (1 > 0).

George does **not** have a single **Dominant Strategy**. His best move is to **match Leo's choice** in every scenario.

Leo's perspective:

- If George chooses **Netflix**, Leo must compare 3 (Netflix) versus 0 (Disney+). Leo's best response is **Netflix** (3 > 0).
- If George chooses **Disney+**, Leo must compare 0 (Netflix) versus 1 (Disney+). Leo's best response is **Disney+** (1 > 0).

Similarly, Leo does **not** have a Dominant Strategy. His best move is also to **match George's choice**.

<u>Multiple Nash Equilibrium</u>

Unlike the Prisoner's Dilemma, this game has two stable outcomes where neither player has an incentive to unilaterally change their choice. These are the **two pure-strategy Nash Equilibria**:

1. **(Netflix, Netflix) (3, 3):** If they are here, neither George nor Leo would choose to switch to Disney+ on their own, as that would immediately drop their payoff from 3 to 0.
2. **(Disney+, Disney+) (1, 1):** If they are here, neither player would choose to switch to Netflix on their own, as that would drop their payoff from 1 to 0.

This coordination challenge illustrates that a game can have **multiple stable outcomes**. The challenge for George and Leo is not deciding *what* to do, but deciding which one of the stable options they should **coordinate** on.

Coordination Game

A Coordination Game is a type of game where players are better off if they coordinate their actions, even if it means compromising (especially when players prefer different things, which we will discuss later).

Unlike the Prisoner's Dilemma, there is no incentive to betray the other player. The challenge is just **picking the same slot**. Whether it is choosing the same streaming service, driving on the same side of the road, or selecting compatible technologies, **coordination ensures mutual benefit**. However, since players make decisions **simultaneously**, without communication and trust, if they might choose **different ones**. That creates another *dilemma*.

GAME TRANSFORMATION

How to Reach the Win-Win outcome?

The core approaches introduced previously can also be applied here to solve the *dilemma* and find a Pareto optimal path: Establishing Cooperation by (1) building trust or (2) modifying the game's structure.

However, since the second method transforms the game into an entirely new structure, the following episodes will focus primarily on **achieving Cooperation by building trust**. This allows us to explore how players navigate while maintain the specific strategic structure of each game type.

Unbalanced Coordination Game

In George and Leo's scenario, there are two pure strategy Nash Equilibrium: (Netflix, Netflix) (3, 3) and (Disney+, Disney+) (1, 1). Both are stable outcomes that possible to happen. However, (3,3) is clearly the Win-Win result, representing Pareto optimal, also a socially optimal outcome *by chance*. To reach this superior equilibrium, players must **cooperate**. By **communicating to build trust**, just like in the Prisoner's Dilemma, they can coordinate their choices toward the most beneficial result for both parties.

If George and Leo talk more and become comfortable with each other, George might admit, "I actually like Netflix more."

To which Leo might laugh and say,

"Really? Me too! Why didn't you just say so earlier?"

Problem solved. They subscribe to Netflix together. Once they agree, the *game disappears.* No more strategies. Just a happy ending.

Balanced Coordination Game

In some Coordination Games, **both equilibrium may have equal total outcomes**, meaning both are Pareto (also socially) optimal. For example, where George and Leo are indifferent between Netflix and Disney+:

		Leo	
		Netflix	Disney+
George	Netflix	**(3, 3)**	(0, 0)
	Disney+	(0, 0)	**(3, 3)**

Figure 1.2.2 Balanced Coordination Game

Now 'which platform is better?' is not the point, because both give the same payoff. The idea is: George and Leo just have to pick the same platform, creating two pure Nash Equilibrium: (Netflix, Netflix) (3, 3) and (Disney+, Disney+) (3, 3).

Coordination Game with Different Preferences

Coordination does not always mean *do the same thing*, it means *work together*. Sometimes, players may prefer different things.

		Leo	
		Netflix	Disney+
George	Netflix	(*3*, **1**)	(0, 0)
	Disney+	(0, 0)	(*1*, **3**)

Figure 1.2.3 Coordination Game with Different Preferences

Let's say George prefers Netflix, but Leo loves Disney+. There are still two Nash Equilibrium: (Netflix, Netflix) (*3*, **1**) and (Disney+, Disney+) (*1*, **3**) Both are stable, but one player always gets more than the other.

To make it fair for both, **compromise** is the key. Through **communication or agreement**, they can build the trust that is necessary to balance things out. However, this level of coordination is typically sustainable only within a **repeated reciprocal game** framework as it requires a *taking turns* strategy.

For example, George and Leo might agree to alternate monthly: Netflix this month, Disney+ next month. This way, both take turns enjoying what they love, try something new, and still split the cost.

Application

Battle of Sexes

		Woman	
		Action movie	Romance movie
Man	Action movie	(*3*, **1**)	(0, 0)
	Romance movie	(0, 0)	(*1*, **3**)

Figure 1.2.4 Action or Romance?

Coordination Game with Different Preferences: This scenario happened in **couple relationships**. One loves action movies, the other prefers romance. If this is repeated reciprocal game, they do not always have to watch Romance or Action; they just have to coordinate by **compromising and taking turns.** (Romance, Romance) this time, then (Action, Action) next time.

Working Life

		Co-worker	
		Tuesday meeting	Wednesday meeting
You	Tuesday meeting	**(3, 3)**	(0, 0)
	Wednesday meeting	(0, 0)	**(3, 3)**

Figure 1.2.5 Tuesday or Wednesday?

Balanced Coordination Game: You and your co-worker need to pick the same date for a team meeting. (Tuesday, Tuesday) or (Wednesday, Wednesday) are equally good. If you choose different dates, the meeting fails. Thus, Coordination is the only goal.

Holiday Plan

		Your Friend	
		Mountain	Beach
You	Mountain	**(1, 1)**	(0, 0)
	Beach	(0, 0)	**(3, 3)**

Figure 1.2.6 Mountain or Beach?

Unbalanced Coordination Game: You and your friend want to go on a holiday trip together. If you two pick the same destination, the trip happens. But if you two pick differently, the trip is cancelled. However, you both prefer the beach to mountain, but no one knows what the other will choose, in static game with no communication. Thus, (Beach, Beach) and (Mountain, Mountain) both are Nash equilibrium.

EPISODE 1.3 - Betty's Night Meal 🐾

The Hawk-Dove Game

It is 2:00 a.m. Betty meows. Again.

Someone needs to get up and feed her.

George hears it.

Leo hears it.

But both pretend to sleep. No one wants to move.

Each wants the other to do it. Because if one gets up, the other can continue to keep sleeping.

The question is: *'Who feeds the cat?'*

GAME MODELING

Betty is hungry!

After successfully coordinating on their streaming service, George and Leo immediately run into conflict over their most important shared responsibility: Betty. When Betty wakes up hungry at midnight, someone must feed her! If both go, they both lose sleep; if neither goes, they both suffer through Betty's meows.

		Leo	
		Feed	Doesn't Feed
George	Feed (Dove)	(3, 3)	**(0, 6)**
	Doesn't Feed (Hawk)	**(6, 0)**	(-1, -1)

Figure 1.3.1 Feed or Doesn't?

<u>The possible outcomes:</u>

- If both choose **Feed**, Betty is fed, but both players wake up and lose rest, resulting in a moderate, inefficient payoff of **(3, 3)**.
- If both choose **Doesn't Feed**, Betty cries all night, and no one gets sleep. The morning starts with an argument, leading to the worst possible mutual payoff of **(-1, -1)**, a *dilemma*.
- If only one person feeds Betty, that person gets the low payoff of **0** (interrupted sleep), while the other enjoys a full night's rest with the high payoff of **6**. This is the most **efficient** outcome for one individual at the expense of the other.

The Search for a Dominant Strategy:

George’s perspective:

- If Leo chooses **Feed**, George's best response is **Doesn't Feed** (6 > 3).
- If Leo chooses **Doesn't Feed**, George's best response is **Feed** (0 > -1).

George does **not** have a single **Dominant Strategy**. His best move is to **do the opposite of Leo**.

Leo’s perspective:

- If George chooses **Feed**, Leo's best response is **Doesn't Feed** (6 > 3).
- If George chooses **Doesn't Feed**, Leo's best response is **Feed** (0 > -1).

Leo also does **not** have a Dominant Strategy. His best move is to **do the opposite of George**.

Multiple Nash Equilibrium

This game has two stable outcomes where neither player has an incentive to unilaterally change their choice. These are the **two pure-strategy Nash Equilibria:**

1. **(Doesn't Feed, Feed) (6, 0):** If Leo is currently feeding, George has the maximum payoff (6) and will not switch. Leo, receiving 0, would love to switch, but if he did (to Doesn't Feed), the payoff would immediately become (-1, -1). Therefore, he is locked in.
2. **(Feed, Doesn't Feed) (0, 6):** Conversely, if George is currently feeding, Leo has the maximum payoff (6) and will not switch. George is locked in at 0 to avoid the mutual chaos of (-1, -1).

In this scenario, each equilibrium is efficient because Betty is fed, but they are **highly unequal**. Each works, but **one player always benefits more by taking advantage of the other’s effort.**

Hawk-Dove Game

The Hawk-Dove game's classic matrix is:

		Animal B	
		Act Dove	Act Hawk
Animal A	Act Dove	(3, 3)	**(1, 5)**
	Act Hawk	**(5, 1)**	(0, 0)

Figure 1.3.2 Hawk-Dove game

Two animals are contesting over a valuable resource. Each player can adopt one of two strategies: Hawk, always fights aggressively for the resource, or Dove, avoids conflict and backs off if the other becomes aggressive.

<u>The possible outcomes:</u>

- If Dove vs Dove, they *share* the resource peacefully which is (Total/2, Total/2), both get (3, 3).
- If Hawk vs Dove, Hawk gets the big reward (5), while Dove gets just a little or nothing (1).
- If Hawk vs Hawk, they fight. Both get hurt and suffer losses due to the fight (0, 0).

Thus, the Nash Equilibrium are **(Hawk, Dove) (5, 1) and (Dove, Hawk) (1, 5).**

The Hawk-Dove Game model represents **conflict and compromise between two players** competing for the same resource. While both benefit from cooperation (acting as Doves), each prefers to let the other bear the cost. The player who acts like a **Hawk will take advantage of the Dove's willingness to cooperate.**

- When two Hawks meet, conflict breaks out, costly for both: If everyone plays Hawk, constant fighting leads to a **dilemma** of mutual loss.

- When two Doves meet, they cooperate and split the reward: However, since neither dominates, each may feel they are giving up more than they get. So, they have an incentive to switch to Hawk, making (Dove, Dove) an unstable outcome.
- When a Hawk meets a Dove, Hawk exploits the Dove's non-aggressive behavior and grabs most of the reward. This creates stable but **unfair** equilibrium: one Hawk, one Dove.

In George and Leo's case, (Feed, Doesn't) (0, 6) and (Doesn't, Feed) (6, 0) are the equilibrium. The cat gets fed, but one player sacrifices comfort for the shared good while the other enjoys uninterrupted sleep. Although **cooperation** (Feed, Feed) (3, 3) is the fairest and **most peaceful** outcome, it is hard to achieve, especially when *neither wants to be the Dove.*

GAME TRANSFORMATION

How to Reach the Win-Win outcome?

To Solve the dilemma and reach the peaceful, Pareto optimal outcome, players must **cooperate**, which requires the establishment of **trust**. Thrust can be established through **communication, agreements, or third-party support.**

The equilibrium of Hawk-Dove is structurally similar to the ones of the Coordination Game with Different Preferences, where one player inevitably receives a higher payoff than the other at any equilibrium point. In such cases, maintaining coordination is sustainable only when the game can be **repeated reciprocal game,** as it requires a *taking turns* strategy.

For example, George and Leo might agree to *alternate* responsibilities: one feeds Betty tonight, the other tomorrow. Or they might co-invest in an automatic feeder, changing the game environment, no more Hawk and Dove, and no one has to wake up at night anymore.

These are fair solutions without needing one to always be the Dove. The problem solved and the *game disappears.*

Application

The *fundamental logic* of the Hawk-Dove game is the opposite of the Coordination game. In a coordination game, both players benefit by choosing the same action and working together. In contrast, in the Hawk-Dove game, players try to avoid choosing the same action. They seek to do the opposite of one another because Hawk always brings a higher individual reward, but if both choose Hawk, it leads to direct conflict, or even "war."

Real-World Example: The Cold War

The Hawk-Dove dynamic is reflected in global politics, where the fear of being taken advantage of makes both start as hawks, neither willing to compromise first. During the Cold War, both the U.S. and the Soviet Union played Hawk, stockpiling nuclear weapons, refusing to back down. But full-scale conflict would destroy both. Mutual fear pushed them to adopt Dove-like behaviors: arms treaties, crisis negotiations, and third-party mediation (e.g. by the UN). However, they just pretend to be Dove to make everyone live peacefully. Since (Dove, Dove) is not a truly stable point, each side still had incentives to act like a Hawk occasionally. Still, by **alternating between Hawk and Dove roles when necessary,** they avoided the worst-case outcome: nuclear war.

Battle of Sexes

		Woman	
		Quiet	Argue
Man	Quiet (Dove)	(2, 2)	**(1, 4)**
	Argue (Hawk)	**(4, 1)**	(-5, -5)

Figure 1.3.3 Quiet or Argue?

Disagreements are normal in relationships. If one partner argues (Hawk) while the other stays quiet (Dove), the one expressing their feelings partner feels heard (4), but the quiet one feels ignored (1). This creates two Pure Strategy Nash Equilibrium: (Argue, Quiet) (4,1) and (Quiet, Argue) (1,4). Both are stable, but one partner (Hawk) always benefits more.

However, if both are Hawk, emotions flare. The result can be serious, even a *breakup* (–5,–5). While both being quiet (2,2) is peaceful, it is unstable, as either partner has an incentive to argue for a better outcome. Thus, if this is repeated reciprocal game, it is better to take turns expressing concerns instead of both argue.

Working Life

		Co-worker	
		Collaborative	Aggressive
You	Collaborative (Dove)	(5, 5)	**(0, 10)**
	Aggressive (Hawk)	**(10, 0)**	(-1, -1)

Figure 1.3.4 Collaborative or Aggressive?

Recently, you and your co-worker are assigned to a new project, subtly competing for the leadership role. You can choose to be:

1. Collaborative (Dove) - sharing responsibilities and making joint decisions
2. Aggressive (Hawk) - taking charge and presenting ideas without consulting

If both are Collaborative (5,5), the project runs smoothly with shared success. However, if one is Aggressive while the other is Collaborative, the Aggressive one gains leadership and high credit (10), leaving the Collaborative one with little (0). If both choose to be Aggressive, the project becomes a battlefield of competing ideas, leading to delays and a dysfunctional team. *The project may fail,* and both have a negative payoff (–1,–1). This game has two Pure Strategy Nash Equilibrium: (Aggressive, Collaborative) (10,0) and (Collaborative, Aggressive) (0,10). The best solution is, when one player prefers to be the "Hawk," the other have to play “Dove.”

Holiday Plan

		Your Friend	
		Plan	Lazy
You	Plan (Dove)	(4, 4)	**(2,** 7**)**
	Lazy (Hawk)	**(**7**, 2)**	(0, 0)

Figure 1.3.5 Plan or Lazy?

After you and your friend pick the same travel destination in the Coordination Game, you are now planning a trip. Both have two choices:

1. Do the planning and handle everything. (Dove)
2. Be lazy and command the other to do the work. Do not take initiative. (Hawk)

If both choose to be lazy and only command others, nothing gets done. *The trip is canceled*, and your companionship is broken. If one person plans while the other stays lazy, the lazy one benefits with no effort, while the planner does all the work.

This version of the Hawk-Dove game shows how Dove's passive behavior can lead to *unfair workloads*, even if *no conflict is visible*. If this is a repeated reciprocal game, you and your friend might take turns being the planner.

EPISODE 1.4 - Betty Doesn't Feel Well… 😿

The Hunter Game

George and Leo notice something is off with Betty, their little cat.

She is not eating, not meowing, barely moves from her blanket fort, and refuses her favorite toy mouse.

George frowns.

"She is never like this. I think we need to take her to the vet."

Leo agrees.

"Yeah, but it's across town, and she screams in the carrier.

We need two people… one to hold her, one to drive."

Both look at each other.

Also, both have a midterm project due soon.

Homework is piling up and the stress is real.

They each silently wonder:

'Will he go if I don't?

'Should I just stay home and get my work done?'

Key:

COORDINATION - Only Get Benefit When Coordinate.

HUNTER - Okay to Be Alone but Can Better Together.

HAWK-DOVE - Want to Take Advantage of Each Other.

GAME MODELING

Taking Betty to the Vet

George and Leo now face a high-stakes coordination challenge. Betty is unwell and requires a veterinary trip. The decision involves a trade-off between securing the best care for Betty (require cooperation, high reward) and completing their individual homework (not require cooperation, safe reward).

		Leo	
		Join Vet Trip	Stay Home
George	Join Vet Trip	**(8, 8)**	(0, 5)
	Stay Home	(5, 0)	**(5, 5)**

Figure 1.4.1 Join Vet Trip or Stay Home?

<u>The possible outcomes:</u>

- If both choose to **Join the Vet Trip**, they achieve the highest mutual payoff of **(8, 8)**. Betty receives help, the trip is efficient with two people, and they still manage to complete their homework with the time saved. This represents the PERFECT, both Pareto and Socially optimal outcome.
- If both choose to **Stay Home**, the outcome is **(5, 5)**. They successfully complete their homework without stress, but Betty remains untreated. This payoff is stable and safe, but ultimately sub-optimal for collective welfare.
- If one joins and the other stays home (e.g. George Joins, Leo Stays), the person who goes alone receives a low payoff of **0** because they struggle or give up on the trip, while the person who stayed receives a low-effort payoff of **5** but has failed to contribute to the collective good.

The Search for a Dominant Strategy:

To analyze the strategic landscape, we determine the best response for each player based on the assumption of rationality. Neither player possesses a **Dominant Strategy** in this game, as their best action depends on the other player's action.

George's perspective:

- If Leo chooses to **Join the Vet Trip**, George must choose between 8 (Join) and 5 (Stay). George's best response is to **Join too** (8 > 5).
- If Leo chooses to **Stay Home**, George must choose between 0 (Join) and 5 (Stay). George's best response is to **Stay too** (5 > 0).

George's rational choice is dependent: his best move is always to match Leo's action.

Leo's perspective:

- If George chooses to **Join the Vet Trip**, Leo must choose between 8 (Join) and 0 (Stay). Leo's best response is to **Join too** (8 > 0).
- If George chooses to **Stay Home**, Leo must choose between 0 (Join) and 5 (Stay). Leo's best response is to **Stay too** (5 > 0).

Similarly, Leo's best move is also to match George's action.

Multiple Nash Equilibrium

This structure results in **two pure-strategy Nash Equilibria**, representing two distinct stable outcomes where neither player has an incentive to unilaterally change their choice:

1. **(Join Vet Trip, Join Vet Trip) (8, 8):** This is both **Pareto and socially optimal** equilibrium. If they successfully coordinate here, any attempt by one player to switch to **Stay Home** would reduce the overall outcome (from 8 to 5).

2. **(Stay Home, Stay Home) (5, 5):** This is the **sub-optimal** equilibrium. If they default to this safe option, neither player will switch to **Join Vet Trip** alone, as that would immediately drop their payoff from 5 to 0.

The strategic problem for George and Leo in this coordination game is how to guarantee that they select the **superior** equilibrium of (8, 8) rather than falling back to the safer, but ultimately inferior equilibrium of (5, 5).

Hunter Game

The Hunter game's classic matrix is:

		Hunter B	
		Hunt Stag	Hunt Hare
Hunter A	Hunt Stag	**(4, 4)**	(0, 3)
	Hunt Hare	(3, 0)	**(3, 3)**

Figure 1.4.2 Stag or Hare?

Hunter A and B are going out to hunt. They can choose between

1. Hunt hare: a smaller animal, but they can catch it *alone*, or…

2. Hunt stag: a big reward, but it requires *cooperation*. One hunter cannot catch it alone. If he tried, he would get nothing, while another who hunt hare still get animal.

The possible outcomes:

- If both hunt stag, this means they cooperate and trust each other, so both get the best result (4, 4).
- If one hunts stag and the other hunts hare, the one who hunts stag gets nothing (0),while the other gets a hare (3).

- If both hunt hare, they get the safe and reliable outcome, but not the best (3, 3).

Nash Equilibrium are **(Stag, Stag) (4, 4) and (Hare, Hare) (3, 3).**

This Hunter Game (also known as the Stag Hunt Game) presents a situation where players have **two types of action**. Both of which can lead to a stable outcome: independence and cooperation.

1. Act independently: get a **safe but not great** result (catch the "hare"), not that great, but acceptable. In George and Leo's scenario, (Stays Home, Stays Home) (5, 5) is stable. No one gets hurt if partner does not cooperate, but no one wins big either.

2. Work together to achieve the best result (catch the "stag") — but *only if both commit.* For example, (Join, Join) (8, 8) is equilibrium, PERFECT Win-Win. However, it is also risky. Because it **only works if both players trust each other** are willing to act. If another does not cooperate, the one who commit it alone suffers: (Join, Stays Home) (0, 5) and (Stays Home, Join) (5, 0). Thus, **neither wants to take the risk alone**.

However, if the **potential reward** from working together is **high enough**, players might be willing to take that risk. In the original Hunter Game, the difference between cooperation and independence is just 1 point: (Hunt Stag, Hunt Stag) (4, 4) compared to (Hunt Hare, Hunt Hare) (3, 3). But in George and Leo's case, the difference is bigger: (Join, Join) (8, 8) and (Stays Home, Stays Home) (5, 5). A 3-point gap might be tempting enough for them to take risk and try choosing "Join", then hope the other does too.

This is **different from the Coordination Game**, where players must work together to win. In a Hunter Game, **cooperation is best, but non-cooperation is still okay**, just not the best. It is about trust and how much players are willing to risk. George and Leo do not *have to* cooperate, however, if they do — they might just save the cat and still finish their homework. That is the *trade-off*. In real life, this Hunter Game shows up everywhere:

- Two friends agree to present a group project… but if one skips out, the other looks bad.
- Two roommates promise to deep clean before inspection… but if one flakes, the other is stuck scrubbing alone.
- Two countries agree to reduce emissions… but if only one follows through, it bears all cost.

GAME TRANSFORMATION

How to Reach the Win-Win outcome?

Just applied the first core approach here: **Establishing Cooperation by building trust through communication.**

However, unlike the Coordination Game with Different Preferences and Hawk-Dove Game, the Hunter Game does **not actually need to be a repeated reciprocal game** to reach a fair outcome as both Nash equilibrium are already *fair*: (Join, Join) (8, 8) and (Stays Home, Stays Home) (5, 5). Players just need to **agree to cooperate.**

The question is simply: *Are they willing to cooperate for the greater reward?*

If George and Leo slow down and talk it through, they might say:
George: "I'll pay for the vet and carry Betty."
Leo: "I'll drive… I know the shortcut since I've lived here longer."
George: "Let's bring our laptops. We can finish homework while we wait."

Problem is solved. Betty gets help and homework gets done. Everyone wins.

Application

Battle of Sexes

		Woman	
		Married	Single
Man	Married	**(6, 6)**	(4, 0)
	Single	(0, 4)	**(4, 4)**

Figure 1.4.3 Married or Single?

Nowadays, people can live happy lives without getting married, but marriage can still bring special benefits like emotional support, and sharing expenses, which help creating financial stability. In this Hunter Game example, if both the man and woman choose to get Married, they each receive (6,6). If one wants to get married but the other does not, the one who wanted marriage feels rejected and gets (0), while the other is fine on their own and gets (4). If both choose to stay Single, they each receive (4,4), living okay alone but missing what a close partnership could bring.

Working Life

		Co-worker	
		Join Team	Solo
You	Join Team	(8, 8)	**(2, 5)**
	Solo	**(5, 2)**	(5, 5)

Figure 1.4.4 Join Team or Solo?

You and a co-worker are competing for an innovation-of-the-year grant, achieving the best outcome for both. If both choose to (Join Team, Join Team) (8,8) and collaborate, you create a strong proposal and win the grant. However, collaboration takes time and trust, and there is a risk the other might not commit, leaving you with a weak proposal (2) while they submit their own solo project and gets (5). If both play it safe and go Solo (5,5), you each get an okay but unremarkable outcome. This game has two Pure Strategy Nash Equilibrium: (Join Team, Join Team) (8,8) and (Solo, Solo) (5,5). While collaborating for the grant is the most rewarding, the "Solo" option offers a safe fallback.

Holiday Plan

		Your Friend	
		Together	Alone
You	Together	**(7, 7)**	(-1, 4)
	Alone	(4, -1)	**(4, 4)**

Figure 1.4.5 Together or Alone?

The last trip in the Hawk-Dove game was successful because you both did the planning together (Dove, Dove).

Now you are looking for another adventure trip. If you and your friend agree to go on the trip (Together, Together), you each enjoy a great trip with shared memories and lower costs (7,7). If you want to go together but your friend cancels later and goes alone, you feel disappointed and waste time or money (gets -1), while your friend has a decent solo trip (gets 4). If both choose to go (Alone, Alone) from the start, they avoid the risk of being let down and have okay trips by themselves, so both get (4,4). Going (Together, Together) is best… but only if both commit.

EPISODE 1.5 - Anywhere But Here 😫

No Pure Equilibrium Game – Mixed Strategy

After solving so many conflicts, George and Leo are now more than just roommates… they are like *brothers*. They study, watch movie, game, and laugh together. Life is smooth. Almost.

But as midterms approach, they need a quiet place to focus. The problem? Betty the cat is too affectionate... always kneading and purring on their keyboards during study time.

So, they leave her with an automatic feeder and move their study sessions to the CCT University Library.

There is just one issue left: Tom.

The library has two zones; the popular window seats, quiet and full of natural light, and the less popular entrance seats, which are noisier.

George and Leo love the window seats. But lately, **Tom and his group of friends** seem to *always* end up sitting nearby — whispering, laughing, distracting.

Even when George and Leo switch to the entrance seats… somehow, Tom still appears.

“Do you think we’ll see them again today?” George sighs.

Leo grumbles, “I just want to read quietly.”

So now, they are stuck with a new game: *how to avoid Tom?*

GAME MODELING

Library Seat Game – Pure Strategy

		Tom & friends	
		Window	Entrance
George & Leo	Window	(0, 5)	(5, 0)
	Entrance	(3, 0)	(0, 3)

Figure 1.5.1 Window or Entrance? : Pure Strategy

<u>The possible outcomes:</u>

- If both groups **choose the same seat**, (Window, Window) (0, 5) or (Entrance, Entrance) (0, 3). George & Leo's study is disturbed, their payoff is 0. Tom's group seems *bizarrely* happy. They prefer the window seat (5) but still enjoy sitting at the entrance (3)… just close to George & Leo is enough!
- If George & Leo **pick a different seat** from Tom's group, (Window, Entrance) (5, 0) or (Entrance, Window) (3, 0). George & Leo get peace and quiet. They are incredibly happy with the window (5), but the entrance (3) is also acceptable — as long as Tom isn't there. Tom's group gains nothing. For some reason, they only enjoy sitting *near* George & Leo.

<u>The Search for a Dominant Strategy:</u>

George & Leo's perspective:

- If they knew Tom would pick Entrance, they would go Window (5 > 0)
- If they knew Tom would pick Window, they would go Entrance (3 > 0)

George & Leo do **not** have a single **Dominant Strategy**. They will try to switch to avoid Tom.

Tom & friends' perspective:

- If they knew George & Leo would pick Entrance, they would go Entrance (5 > 0)
- If they knew George & Leo would pick Window, they would go Window (3 > 0)

Tom & friend do **not** have a single **Dominant Strategy**. Oddly, Tom just wants to follow them.

Pure-Strategy Nash Equilibrium

This game has **no pure strategy Nash equilibrium**, because each group wants a different outcome. **The only solution is Mixed Strategy Nash Equilibrium: both sides must randomize their choices to stay unpredictable.**

Library Seat Game – Mixed Strategy

To find the mixed strategy Nash equilibrium, we assume:

- Let **p** be the probability that *George & Leo* choose the **Window** seat.
- Let **q** be the probability that *Tom & Friends* choose the **Window** seat.

		Tom & friends	
		Window (q)	Entrance (1-q)
George & Leo	Window (p)	(0, 5)	(5, 0)
	Entrance (1-p)	(3, 0)	(0, 3)

Figure 1.5.2 Window or Entrance?˙: Mixed Strategy

Step 1: Make G&L feel Indifferent Between Their Two Strategies

To make *G&L* feel indifferent between Window and Entrance, Tom will choose his own probabilities (q for Window and 1-q for Entrance) in such a way that G&L's expected payoff (G&L average reward over many plays) from picking Window is exactly equal to their expected payoff from picking Entrance. This means Tom's randomization will make G&L unable to prefer Window over Entrance or Entrance over Window in the long run because now G&L cannot guess Tom's moves!

To find Tom's probability (q), we set **expected payoffs** for *G&L* choosing Window and Entrance *equal*:

Expected Payoff = Σ (Payoff × Probability the other plays corresponding move)

		Tom & friends	
		Window (q)	Entrance (1-q)
George & Leo	Window (p)	(0, 5)	(5, 0)
	Entrance (1-p)	(3, 0)	(0, 3)

Figure 1.5.3 Window or Entrance? : Mixed Strategy (Continued)

- Expected payoff if G&L pick **Window**: $(0 \times q) + (5 \times (1-q))$
- Expected payoff if G&L pick **Entrance**: $(3 \times q) + (0 \times (1-q))$

$$0 \times q + 5 \times (1-q) = 3 \times q + 0 \times (1-q)$$
$$5 - 5q = 3q$$
$$q = 5/8 \text{ (Tom's probability)}$$

<u>Step 2: Make Tom & Friends Indifferent Between Their Two Strategies</u>

We set expected payoffs for *Tom* choosing Window and Entrance *equal* because now Tom cannot guess G&L's moves:

- Expected payoff if Tom pick **Window**: $(0 \times p) + (3 \times (1-p))$
- Expected payoff if Tom pick **Entrance**: $(5 \times p) + (0 \times (1-p))$

$$3(1 - p) = 5p$$
$$3 - 3p = 5p$$
$$p = 3/8 \text{ (G\&L's probability)}$$

<u>Final Answer – Mixed Strategy Nash Equilibrium</u>

George & Leo choose **Window** with probability (p) **3/8, Entrance** with **5/8**. *Tom & Friends* choose **Window** with probability (q) **5/8,** and **Entrance** with **3/8**.

Mixed Strategy

A mixed strategy is when a player randomizes their choices, assigning a probability to each possible action. This approach is necessary when no single pure strategy guarantees a best outcome for any player.

In George & Leo and Tom's group's library seating game, George & Leo originally wanted to avoid Tom, while Tom's group wanted to follow them. Since no slot could satisfy both groups simultaneously, a mixed strategy had to be used. Players would randomly choose their spot so others could not guess their move, causing chaos. The example is that George & Leo would randomly sit to avoid being followed. While Tom's group would randomly sit to make it hard for George & Leo to run away. So, **both groups end up feeling indifferent between choosing the Window and Entrance seats as now their opponent is unpredictable.**

However, once their intentions were clear, the game changed. They now fully coordinate: pure strategy Nash equilibrium where both pick window and get the happy ending.

When is a Mixed Strategy Used?

Mixed strategy usually happens in games where:

1. No pure strategy Nash equilibrium exists.

This is like George Leo and Tom's case, where one player completely dislikes what the other prefers. Their choices are in complete conflict, so a pure strategy Nash equilibrium cannot be found. In such cases, players can only use a mixed strategy to solve the game. The most classic example is Matching Pennies, which has *no pure strategy Nash equilibrium* at all:

		Player B	
		Head (q)	Tail (1-q)
Player A	Head (p)	(1, -1)	(-1, 1)
	Tail (1-p)	(-1, 1)	(1, -1)

Figure 1.5.4 Matching Pennies

Two players, A and B, simultaneously choose **Head** or **Tail** of the penny. A holds the first penny, B holds the second one. If both players match (both choose Heads or both choose Tails), **A wins** (+1) while **B loses** (–1). If they differ, **B wins** (+1) while **A loses** (–1). We would see that **A wants them to be a match, while B wants a difference.**

To find the mixed strategy Nash equilibrium, there are two main steps.

Step 1: Make Player A indifferent between Heads and Tails

Expected payoff if A chooses Head = Expected payoff if A chooses Tail

$$1q+(-1)(1-q) = -1q+(1)(1-q)$$

$$2q-1 = -2q+1$$

$$\text{So, } q = 1/2$$

Step 2: Make Player B indifferent between Heads and Tails

Expected payoff if B chooses Head = Expected payoff if B chooses Tail

$$-1p+(1)(1-p) = 1p+(-1)(1-p)$$

$$\text{So, } p = 1/2$$

Mixed Strategy Nash Equilibrium:

- A Plays Heads with probability 1/2, Tails with 1/2.
- B Plays Heads with probability 1/2, Tails with 1/2.

Matching Pennies is a **zero-sum game** (A game where one player's gain is exactly the other's loss. Their payoffs always sum to zero.) In this type of game, there is **always** no pure strategy Nash equilibrium.

However, mixed strategy games are *not* only used in zero-sum games. For example, George & Leo and Tom's case is **not a zero-sum game**, but their choices are in conflict, which also causes no pure strategy Nash equilibrium. A mixed strategy simply means a player randomizes their choices because no pure strategy guarantees a best outcome for both. Thus, **it can be used in any game where a pure strategy Nash equilibrium does not exist**, no matter if it is zero-sum or not.

2. Multiple Equilibrium Games.

Mixed strategies can also be used in games that have multiple equilibrium. This means "A game may have both pure strategy and mixed strategy Nash equilibrium."

In fair games, such as the Balanced Coordination Game, the payoffs for each player in each equilibrium are **equal**. If players have no reason to prefer one over the other, randomizing equally becomes a rational choice. The probability for each equilibrium will be split equally, often at 1/2. This also reflects in the **highly symmetric game** like Matching Pennies.

However, **in unfair games (or less symmetric game)** like Unbalanced Coordination, Coordination with Different Preferences, and Hawk-Dove, the equilibrium payoffs are **unequal**. We cannot simply predict which equilibrium will happen or assume players will randomize equally. In these cases, we use a mixed strategy to find the exact probabilities that make each player indifferent between their available strategies. This calculation helps us understand which equilibrium has a higher chance of occurring.

Even though each player's choice seems unpredictable, **a mixed strategy helps us to calculate the probability of each potential outcome**, giving us a clearer picture of the game's likely result.

✓ Balanced Coordination (Fair or symmetric game):

		Leo	
		Netflix (q=1/2)	Disney+ (1-q=1/2)
George	Netflix (p=1/2)	**(3, 3)**	(0, 0)
	Disney+ (1-p=1/2)	(0, 0)	**(3, 3)**

Figure 1.5.5 Balanced Coordination : Mixed Strategy

Let p be the probability that George chooses Netflix.

Let q be the probability that Leo chooses Netflix.

<u>To make George feels indifferent between Netflix and Disney+:</u>

$$3q+0(1-q) = 0q+3(1-q)$$

$$3q = 3-3q$$

$$\text{So, } q = 1/2$$

<u>To make Leo feels indifferent between Netflix and Disney+:</u>

$$3p+0(1-p) = 0p+3(1-p)$$

$$\text{So, } p = 1/2$$

Thus, in addition to the two pure strategy Nash equilibrium at (Netflix, Netflix) (3,3) and (Disney+, Disney+) (1,1), there is a mixed strategy Nash equilibrium:

- George chooses Netflix with probability 1/2, and Disney+ with probability 1/2.
- Leo chooses Netflix with probability 1/2, and Disney+ with probability 1/2.

However, when we look at **a mixed strategy in a multiple equilibrium game** like the Balanced Coordination Game (where both players benefit only if they coordinate: (Netflix, Netflix) or (Disney+, Disney+)), it's important to understand what it tells us.

While the pure strategy gives us two stable points at coordinated pairs (Netflix, Netflix) and (Disney+, Disney+), a mixed strategy does not guarantee players will coordinate. Instead, **it tells us the "probabilities" each player will choose each choice** (e.g. George picks Netflix with 1/2 probability, Leo picks Netflix with 1/2 probability). Under a mixed strategy setting, George and Leo might end up in an uncoordinated outcome like (Netflix, Disney+) (1/2, 1/2) or (Disney+, Netflix) (1/2, 1/2).

In conclusion, a mixed strategy gives us *just the possibility of coordination*, but **it does not guarantee players will coordinate on the same choice**, unlike the pure strategy equilibrium which are themselves coordinated outcomes.

✓ Unbalanced Coordination (Unfair or less symmetric game):

		Leo	
		Netflix (q=1/4)	Disney+ (1-q=3/4)
George	Netflix (p=1/4)	**(3, 3)**	(0, 0)
	Disney+ (1-p=3/4)	(0, 0)	**(1, 1)**

Figure 1.5.6 Unbalanced Coordination : Mixed Strategy

Let p be the probability that George chooses Netflix movie.

Let q be the probability that Leo chooses Netflix movie.

<u>To make George feels indifferent between Netflix and Disney+:</u>

$$3q+0(1-q) = 0q+1(1-q)$$

$$3q = 1-q$$

$$\text{So, } q = 1/4$$

<u>To make Leo feels indifferent between Netflix and Disney+:</u>

$$3p+0(1-p) = 0p+1(1-p)$$

$$3p = 1-p$$

$$\text{So, } p = 1/4$$

Thus, in addition to the two pure strategy Nash equilibrium at (Netflix, Netflix) (3,3) and (Disney+, Disney+) (1,1), there is a mixed strategy Nash equilibrium:

- George chooses Netflix with probability 1/4, and Disney+ with probability 3/4.
- Leo chooses Netflix with probability 1/4, and Disney+ with probability 3/4.

There is a question raised, **if Disney+'s payoff is moved down lower than before, will the probability that players choose Disney+ lower too?** The answer is *NO.* In a mixed strategy, a player's probabilities are set to make their opponent feel indifferent, not to reflect their own payoff preferences.

Think back to the Library Seat conflict: George & Leo (Player A) randomize their seat to make Tom (Player B) unable to predict their move and feel indifferent between Tom's own choices *("Whatever I choose is the same because I don't know which seat they will pick," Tom think.)* **George & Leo are not choosing their seats to get the highest payoff themselves, but to keep Tom guessing.**

By comparing a Balanced Coordination (Netflix and Disney are equally preferred) with an Unbalanced Coordination (Disney is less appealing), we might expect the probability of George choosing Disney to go down.

However, **it actually goes up, because George must now play Disney+ more often to make Leo feel indifferent between Leo's two choices.** The probabilities (p and 1-p) are adjusted to keep George's opponent (Leo) guessing and ensure that Leo will have no single best choice.

✓ Coordination with Different Preference (Unfair or less symmetric game):

		Woman	
		Action movie (q=1/4)	Romance movie (1-q=3/4)
Man	Action movie (p=3/4)	(**3**, *1*)	(0, 0)
	Romance movie (1-p=1/4)	(0, 0)	(*1*, **3**)

Figure 1.5.7 Coordination with Different Preference : Mixed Strategy

Let *p* be the probability that the man chooses Action movie.

Let *q* be the probability that the woman chooses Action movie.

<u>To make the man feels indifferent between Action and Romance:</u>

$$3q+0(1-q) = 0q+1(1-q)$$

$$3q = 1-q$$

$$\text{So, } q = 1/4$$

<u>To make the woman feels indifferent between Action and Romance:</u>

$$1p+0(1-p) = 0p+3(1-p)$$

$$1p = 3-3p$$

$$\text{So, } p = 3/4$$

In conclusion, this game has two pure equilibrium at (Action, Action) (3,1) and (Romance, Romance) (1,3), including a mixed equilibrium at (p, q) (3/4, 1/4).

✓ Hawk-Dove (Unfair or less symmetric game):

		Leo	
		Feed (q=1/4)	Doesn't Feed (1-q=3/4)
George	Feed (Dove) (p=1/4)	(3, 3)	**(0, 6)**
	Doesn't Feed (Hawk) (1-p=3/4)	**(6, 0)**	(-1, -1)

Figure 1.5.8 Hawk-Dove game : Mixed Strategy

Let *p* be the probability that George chooses Dove.

Let *q* be the probability that Leo chooses Dove.

<u>To make George feels indifferent between Dove and Hawk:</u>

$$3q+0(1-q) = 6q+(-1)(1-q)$$

$$3q = 6q-1+q$$

$$\text{So, } q = 1/4$$

<u>To make Leo feels indifferent between Dove and Hawk:</u>

$$3p+0(1-p) = 6p+(-1)(1-p)$$

$$3q = 6q-1+q$$

$$\text{So, } p = 1/4$$

In conclusion, this game has two pure equilibrium at (Feed, Doesn't Feed) (0,6) and (Doesn't Feed, Feed) (6,0), including a mixed equilibrium at (p, q) (1/4, 1/4).

As mentioned in the Balanced Coordination (Netflix & Disney+) mixed strategy game, a **pure strategy equilibrium** guarantees a specific, stable outcome, such as the (Hawk, Dove) and (Dove, Hawk) pairs in this case. In contrast, a **mixed strategy equilibrium** gives us a set of probabilities for each player's actions. It does not guarantee a specific outcome in a single play, which means it is possible for two players to choose "Hawk" simultaneously.

In the Hawk-Dove game, the pure strategy guarantees a conflict-free results of one Hawk and one Dove, **the mixed strategy creates a significant chance of a mutually destructive *war* outcome.** The mixed strategy equilibrium tells us that players, by trying to be unpredictable, risk this negative result, which is why pure strategies (when they exist and are mutually beneficial) are often preferred.

3. Players want to keep their opponent uncertain by acting randomly.

Imagine a company deciding whether to launch a new product early or delay to make it better. Its competitor is also deciding whether to copy the product and sell it first or release something totally new. If the competitor knew the first company would delay, it might quickly copy and sell the product now. But if the first company launched early, the competitor might waste time trying to copy something already on the market. So, both companies want to keep each other guessing. They might randomly choose their strategies to avoid being predictable (e.g. a 2/3 chance to launch now, and a 1/3 chance to delay).

4. Each player's best move depends on what the other might do.

This is also like George & Leo and Tom's case. George & Leo prefer the window seats, and Tom's group prefers them too. But George & Leo only want to sit by themselves, while Tom's group wants to sit near them. So, George & Leo's best move depends on where Tom is likely to go. If Tom's group gets the window, even though George & Leo love the window more, they will switch to the entrance without hesitation to avoid Tom! This makes Tom want to randomly choose his seat so George & Leo cannot guess his moves.

GAME TRANSFORMATION

How to Reach the Win-Win outcome?

In previous episodes, we focused on maintaining the specific structure of a game, such as ensuring a Hunter game remained a Hunter game, by applying our first core approach to solve the dilemma: **Establishing Cooperation through trust**.

In this episode, we will shift our focus to the second method: **Changing the game's environments to support player cooperation,** exploring how modifying the environment can lead to mutual success and resolve **a strategic complexity dilemma** at its source.

For example, *what if we reveal that Tom and his friends do not mean to disturb George and Leo, but actually just want to make friends?* By recontextualizing the situation, the perceived *ingredients* of the conflict change, transforming antagonistic interactions into opportunities to cooperation.

After a few days of dodging and switching seats, George and Leo were getting tired of running away.

Then something unexpected happened.

As they walked into the library one afternoon, Tom looked up with a grin and called out, "Hey, George! Leo! Wanna join our football team this weekend? We're short two players… and you guys look like you could handle it."

George blinked in surprise.

So that's why Tom's group kept sitting nearby... *they just wanted to be friends.*

George let out a half-laugh, still a little suspicious. “Midterms first. We’re barely surviving.”

Leo nodded. “Ask us again after the exam.”

Tom chuckled. “Fair enough. Hey, while we’re at it… mind if we join your study group? We’re kinda lost in Game Theory.”

George glanced at Leo, then back at Tom’s group. Their faces looked genuinely desperate.

Finally, George sighed and smiled. “Sure… just no whispering this time.”

“Deal!” Tom grinned, and his friends bumped fists in celebration.

That day, both groups choose the window seat with probability of 1 — not to annoy each other, but to cooperate and study.

Once their intentions were clear, the game *changed*. They now fully cooperate: **pure strategy Nash equilibrium where both pick window and get the happy ending**.

Application

Battle of Sexes

		Woman	
		Expect Rose (q)	Expect Jewelry (1-q)
Man	Give Rose (p)	**(-5, 5)**	*(10, -10)*
	Give Jewelry (1-p)	*(5, -5)*	**(-10, 10)**

Figure 1.5.9 Rose or Jewelry?

The Gift-Giving Game is a scenario about surprise and how value is perceived. The man wants to surprise his woman by giving a gift that exceeds her expectations, while the woman hopes the gift matches what she expects.

- **(Give Rose, Expect Rose):** The man feels bad (–5) because he put effort into choosing the rose, but failed to surprise her since she expected it. The woman, however, feels moderately happy (5) because her expectation was met; while it is a pleasant gift, its lower value compared to jewelry means her satisfaction is not at its peak.
- **(Give Rose, Expect Jewelry):** The man feels he "won" because he successfully surprised her without a large financial cost (10). On the other hand, the woman is very disappointed (–10) because she expected a high-value jewelry but received a rose.
- **(Give Jewelry, Expect Rose):** The man successfully surprised her, but the jewelry was a bit expensive for his budget (5). The woman is slightly disappointed (-5) because she just wanted roses, however, since the jewelry is expensive and she received it as a gift, her disappointment isn't severe.
- **(Give Jewelry, Expect Jewelry):** The man feels frustrated (–10) because he spent a lot of money on jewelry, but he cannot surprise her. The woman, however, feels she "won" (10) because her high expectations were perfectly met.

Man's perspective:

- *If the woman expects Rose, the man would give Jewelry (5 > -5).*
- *If the woman expects Jewelry, the man would give Rose (10 > -10).*

Woman's perspective:

- **If the man gives Rose, the woman would expect Rose (5 > -10).**
- **If the man gives Jewelry, the woman would expect Jewelry (10 > -5).**

This is a **zero-sum game** where players' desires conflict: the man wants to surprise, the woman wants her expectation met. Because of this conflict, no pure strategy Nash Equilibrium exists. The only rational choice for both players is to randomize their actions, he tries to keep his gift choice secret, and she tries to keep her true desires hidden — creating a mixed-strategy equilibrium:

Let p be the probability that man Gives Rose.

Let q be the probability that woman Expects Rose.

To make the man feels indifferent between Rose and Jewelry:

$$-5q+10(1-q) = 5q+(-10)(1-q)$$

$$\text{So, } q = 2/3$$

To make the woman feels indifferent between Rose and Jewelry:

$$5q+(-5)(1-q) = -10q+10(1-q)$$

$$\text{So, } p = 1/2$$

In conclusion, the mixed-strategy Nash equilibrium in this game is (p, q) (1/2, 2/3): the man Gives Rose with probability 1/2, and Gives Jewelry with probability 1/2. The woman Expects Rose with probability 2/3, and Expects Jewelry with probability 1/3.

Working Life

		Co-worker (Marketing)	
		Prep for Innovation (q)	Prep for Feasibility (1-q)
You (Product)	Push Innovation (p)	**(0, 0)**	*(10, -10)*
	Push Feasibility (1-p)	*(5, -5)*	**(0, 0)**

Figure 1.5.10 Innovation or Feasibility?

This is a Project Pitch Game. You (the Product Lead) and your co-worker (the Marketing Lead) are giving a project presentation. You each choose a strategy:

1. Innovation: Focus on new, groundbreaking ideas. This is high-risk, high-reward features.
2. Feasibility: Focus on practical, solid data. This guaranteed return on investment.

The success depends on how well the project is received, but you both also compete for individual recognition. You want to surprise your co-worker to get a better score (the Offense), while your co-worker wants to guess your strategy to avoid looking bad (the Defense). This conflict means there is no single best choice for either of you.

- **(Push Innovation, Prep for Feasibility):** You surprise them with "Innovation," you get a huge win (10). Co-worker 's practical marketing plan seems mismatched and uninspired (-10).
- **(Push Feasibility, Prep for Innovation):** you surprise them with "Feasibility," reliable one with all the data. You get a smaller but still good win (5). Co-worker's flashy marketing campaign seems out of touch with the project's core strengths (-5).
- **(Push Feasibility, Prep for Feasibility) or (Push Innovation, Prep for Innovation):** If you both choose the same strategy, your work is a perfect match. The presentation is coherent but predictable. Neither of you stands out, so no one gains or loses individual credit (0, 0).

Your perspective (Product Lead):

- *If Co-worker prepares for Innovation, you would Push Feasibility (5 > 0).*
- *If Co-worker prepares for Feasibility, you would Push Innovation (10 > 0).*

Co-worker's perspective (Marketing Lead):

- **If you push Innovation, Co-worker would Prepare for Innovation (0 > –10).**
- **If you push Feasibility, Co-worker would Prepare for Feasibility (0 > –5).**

You always want to surprise Co-worker, while your co-worker always wants to match you to *neutralize* his/her advantage. Because of this conflict, no pure strategy Nash Equilibrium exists. The only solution is Mixed Strategy, where you both randomize your moves to be unpredictable:

Let p be the probability that you Pushes Innovation.

Let q be the probability that Co-worker Preps for Innovation.

To make you feel indifferent between Pushing Innovation and Feasibility:

$$0(q)+10(1-q) = 5(q)+0(1-q)$$

$$\text{So, } q = 2/3$$

To make Co-worker feel indifferent between Prepping for Innovation and Prepping for Feasibility:

$$0(p)+(-5)(1-p) = (-10)(p)+0(1-p)$$

$$\text{So, } p = 1/3$$

In conclusion, the mixed-strategy Nash equilibrium in this game is (p, q) = (1/3, 2/3): you Push Innovation with probability 1/3, and Push Feasibility with probability 2/3. Your co-worker Preps for Innovation with probability 2/3, and Preps for Feasibility with probability 1/3. You would see that, **even though "Pushing Innovation" is a powerful strategy that offers you a big reward (10), you use it less often than "Pushing Feasibility (5)."** This is because you want to make your co-worker unable to guess your strategy. By being unpredictable, you force your co-worker to feel indifferent between his/her own choices, which prevents him/her from gaining an advantage over you.

Holiday Plan

		The couple	
		11.30 (q)	13.30 (1-q)
You & Your Friend	11.30 (p)	**(2, 2)**	*(0, 6)*
	13.30 (1-p)	*(1, 5)*	**(3, 3)**

Figure 1.5.11 11.30 AM or 13.30 PM?

Perfect time for both of you to go to Disneyland! You and your friend are at a Disney restaurant, and there are only two lunch times left: 11:30 AM (a regular meal) or 13:30 PM (a special princess meal). Another couple is also trying to decide.

The conflict here is that if you both choose the same time, you will have to share the meal. But if you choose different times, your group will get a full meal, and the other couple will also get a full meal and free drinks. This is because there is a special promotion for couples, but since your group are friends, you do not get the free drinks.

- **(11.30, 11.30):** Both groups share the regular meal. Neither group gets a special advantage (2,2).
- **(13.30, 13.30):** Both groups share the princess's special meal. You both get a better meal, but no one gets an individual edge (4,4).
- **(11.30, 13.30):** The couple gets the special meal and free drinks (6). You feel it is unfair and only get the regular meal (0).
- **(13.30, 11.30):** The couple gets free drinks but only the regular meal (5). You get the special meal but no free drinks (1).

Your group's perspective:

- *If couple choose 11.30 AM, you would choose 11.30 PM (2 > 0).*
- *If couple chooses 13.30 PM, you would choose 13.30 AM (3 > 1).*

The couple's perspective:

- **If you and your friend choose 11.30, they would choose 13.30 AM (6 > 2).**
- **If you and your friend choose 13.30 PM, they would choose 11.30 PM (5 > 3).**

This is not a zero-sum, but it has no pure strategy equilibrium. Player preferences are conflict to each other: your group always wants to coordinate with the couple to avoid them getting free drinks, while the couple always wants to separate from your group to get the free drinks. The only solution is Mixed Strategy, where you both randomize your moves to be unpredictable:

Let *p* be the probability that you and friend to choose 11.30 AM.

Let *q* be the probability that couple to choose 11.30 PM.

To make you indifferent between its two strategies:

$$2(q)+0(1-q) = 1(q)+3(1-q)$$

$$\text{So, } q = 3/4$$

To make the couple indifferent between its two strategies:

$$2(p)+5(1-p) = 6(p)+3(1-p)$$

$$\text{So, } p = 1/3.$$

In conclusion, the mixed-strategy Nash equilibrium in this game is (p, q) (1/3, you're your group choose 11.30 AM with probability 1/3, and choose 13.30 PM with probability 2/3. The couple choose 11.30 AM with probability 3/4, and choose 13.30 PM with probability 1/4.

EPISODE 1.6 - To the Space Mountain 🚀 !

Network Traffic Game

Thanks to a mountain of Pepsi bottles, George and Leo score a FREE trip to Disneyland.

Neither of them has ever been here before, so naturally, they rely on what everyone else online says: “Go to Space Mountain first. It’s the best ride!”

So that’s the plan. But they aren’t the only ones with that idea.

On average, about 100 people — **100 players** head for Space Mountain each day, George and Leo included.

The park map shows more than one route to get there. Some paths are short, others longer, but the real kicker is:

If too many people pile onto the same route, it gets crowded and slows down.

If traffic spreads out, everyone gets there faster.

George studies the map.

Leo frowns.

“Which way should we go?”

The game is on: everyone’s goal is to *reach Space Mountain in the shortest possible time*… but **the outcome depends on what all the other visitors decide too.**


Disneyland
INFORMATION
Space Mountain
Which way should we go?

GAME MODELING

Which way should we go?

George and Leo must get from the Entrance (A) to Space Mountain (B).

At the entrance A, they see two long lines of people waiting:

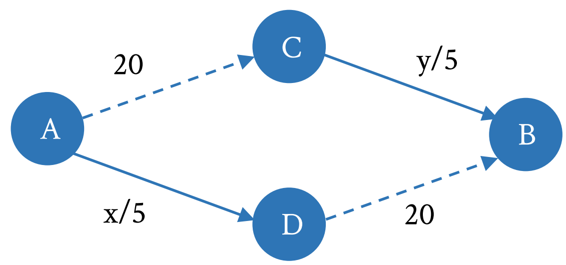


Figure 1.6.1 Transportation Network: From node A to B

1. First Line ($A \rightarrow C \rightarrow B$): This line begins at spot C and stretches to B. First, Sara and George must walk 20 minutes from A to C just to get into this line. If y people are already in front of them, they must wait y/5 minutes at C before reaching B.
2. Second Line ($A \rightarrow D \rightarrow B$): This line starts from Entrance A and stretches to D. If x people are already in front of them, they must wait x/5 minutes at A before reaching D. After getting their turn at D, they still need to walk 20 more minutes to reach B.

Thus, the Space Mountain Possible Routes:

1. $A \rightarrow C \rightarrow B$: Cost of travel = 20 + (y/5) minutes, where y is the number of people in the CB line.
2. $A \rightarrow D \rightarrow B$: Cost of travel = (x/5) + 20 minutes, where x is the number of people in the AD line.

When there are routes with different travel times, all **100 players** — *not just two players anymore* — have an incentive to switch from the slower route to the faster one. Therefore, at Nash Equilibrium, both routes take the same total travel time:

$$(x/5) + 20 = 20 + (y/5)$$

Since there are 100 people trying to get to Space Mountain B every day:

$$x = y \text{ and } x + y = 100$$

Solving gives: $x = 50$, $y = 50$

Traffic at Equilibrium

- Both routes take equal time: (50/5) + 20 = **30 minutes.**
- Total cost-of-travel for all visitors: 30 * 100 = 3,000 minutes

No matter which route George and Leo choose, they must spend 30 minutes to get on the ride.

Adding a New Route! *(Heavy Traffic Case)*

Continuing from Scenario 1, Disneyland announces a new FastPass route, allowing visitors to skip 20 minutes of walking by using a short tunnel connected from D to C. This creates a third option for visitors:

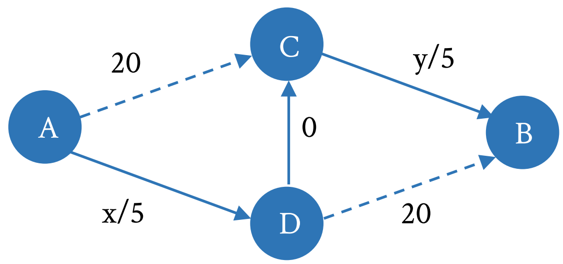


Figure 1.6.2 Adding a new edge in Heavy Traffic Case

Now, there are 3 possible routes:

1. A → C → B : Cost of travel = 20 + (y/5) minutes.
2. A → D → B : Cost of travel = (x/5) + 20 minutes.
3. A → C → D → B (The new FastPass route): FastPass QR tickets are only available at the Entrance A. Since it is under a trial period, visitors can get a QR ticket for free now. Visitors can skip the long and tiring 20-minutes of walking at AC and DB.

   After waiting in line at A to get their turn at D, they can use a direct shortcut tunnel from D to C, which takes just a second, like a space hole transport! After that, they then wait in line at C until they get their turn at B. So, Cost of travel = (x/5) + 0 + (y/5)

Since all **100 visitors** want to reduce their own time, everyone will choose the new, *seemingly* fastest way, which is DC (cost 0). This creates a new Nash Equilibrium where everyone uses the new route: x = 100, y = 100.

A Traffic at Equilibrium

- All **100 players** on the A → C → D → B will now take: (100/5) + (100/5) = **40 minutes**, which is an **increase from the 30 minutes in Scenario 1.**
- Total cost-of-travel for all visitors: 40 * 100 = 4,000 minutes
- Now, the original routes (A → C → B and A → D → B) also require 20 + (100/5) = 40 minutes, meaning the entire traffic network's efficiency decreases.

This is a perfect example of **Braess's Paradox**, where adding a new route to a network can actually lead to a worse outcome for everyone.

It Rains *(Small Traffic Case)*

What if the weather forecast predicts a big rain, causing **only 50 visitors** to go to Space Mountain that day?

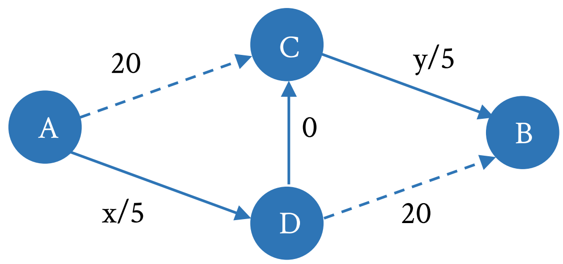


Figure 1.6.3 Adding a new edge in Small Traffic Case

There are 3 possible routes:

1. A → C → B : Cost of travel = 20 + (y/5) minutes.
2. A → D → B : Cost of travel = (x/5) + 20 minutes.
3. A → C → D → B (New FastPass route) : Cost of travel = (x/5) + 0 + (y/5)

Since all **50 visitors** want to reduce their own time, everyone will choose the new, *seemingly* fastest way, which is DC (cost 0). This creates a Nash Equilibrium where everyone uses the new route: x = 50, y = 50

A Traffic at Equilibrium

- All **50 visitors** on the A → C → D → B will now take: (50/5) + (50/5) = **20 minutes**, which is a **decrease from the 30 minutes in Scenario 1.**
- Total cost-of-travel for all visitors: 20 * 50 = 1,000 minutes

A Traffic at Equilibrium (continued)

- The original routes (A → C → B and A → D → B) still require $20 + (50/5) = 30$ minutes to travel, which is the same as the time in Scenario 1. Everyone on the new A → C → D → B route has no incentive to switch back to the original routes because their travel time of 20 minutes is already the fastest option.

This shows that the new FastPass route **can actually improve the network if the number of players is not high enough** to create congestion. The outcome of adding a new route is highly dependent on the number of players.

However, a player does not know the exact number of people who will use the new route. As a result, every time they choose this new option, they face a gamble: **either** they will get a **faster outcome, or** their collective choices will lead to the **negative outcome of Braess's Paradox.** Both are possible.

Braess's Paradox

Braess's Paradox is a counterintuitive finding in network theory that shows **adding a new route to a network can increase the total travel time for everyone.**

The paradox occurs because when a new, seemingly faster shortcut is introduced, individuals act in their own self-interest. They all rush to take the new route, thinking it will save them time. However, this collective behavior leads to overcrowding on the new path and related routes, making the entire system less efficient and increasing the travel time for all users, including those who did not even use the new route.

In simple terms, **what is good for the individual is not always good for the group.** The overall network becomes less efficient when individuals make decisions based on what they perceive to be their best option.

GAME TRANSFORMATION

More Choices: Problem or Chance?

The traffic game is an example of another dilemma: more choices do not always lead to a better outcome. While we often believe that more options are *always* beneficial, they can *sometimes* lead to a worse outcome, creating a **dilemma** for all players involved.

Although the network traffic game is not represented in a matrix like other static games we have discussed, the underlying idea is just like transforming from a static game with a few players and choices (like a 2x2 matrix) to a larger number of players and choices, like n travelers.

In the Disneyland traffic game, all visitors initially have two routes to choose from. After adding a new route, the travel time can be increased instead of decreasing congestion on the original routes if there are too many players in the network. This is because everyone chooses the new option, hoping it will give them a better payoff, but they do not consider the collective impact of their actions.

This new choice is like a mystery box, represented by the "?" in the matrix below: no one knows what is inside. It might lead to a better outcome, but it can also lead to a special case of **Braess's Paradox**, where a new option makes the final result worse. If both firms choose it, **it can be a success for both... or a mutual dilemma.**

	The Original Equilibrium	Firm B		
		Producing A	Producing B	Wait and see
Firm A	Producing A	**(2, 4)**	(3, 2)	(?, ?)
	Producing B	(1, 2)	(2, 0)	(?, ?)
	Wait and see	(?, ?)	(?, ?)	(?, ?)

Figure 1.6.4 Dilemma of More Choices

This can be applied to real-life situations. For example, consider a small village where only 10 people have a high-level education. This small group can choose high-skilled jobs with high salaries because they face little competition.

But what happens if everyone in the village gets a high-level education? Now, everyone has more career options, but the number of high-paying jobs remains limited. Companies might see this as an opportunity to depress wages as many people compete for a few high-level positions. While some people will still get these jobs, their salaries may be lower than they would have been before. Others, despite their education, may have to take lower-skilled jobs. The overall outcome for the group is worse, even though every individual has more choices.

Application

Real-World Example: New Version of War

Braess's Paradox is not limited to traffic or job markets. It can be applied to complex global issues, like the development of nuclear or biological weapons.

Long ago, countries used only guns and knives to fight, but after nuclear weapons emerged, a two-country war turned into a dilemma for everyone. Similarly, the introduction of biological weapons — another "choice" in warfare has created a new, more terrible dilemma for the entire world. The network traffic game is just a simple model that demonstrates this powerful idea: more choices can sometimes lead to a worse outcome for all players.

Season 2: Dynamic Games

In these games, players move in a **sequence**, not at the same time. Unlike static games, where everyone moves simultaneously, a dynamic game involves **turn-taking**, like board games or card games. One player moves first, and the second player observes the first player's choice before making their own response. Because of this, a player's decision is always informed by the choices that were made before their turn. They have specific information about the past, and that helps them see: the choices or *opportunities* they have now based on previous moves, and the payoffs they can expect at the end of the game.

EPISODE 2.1 - Open House Booth Battle ⚔️

Basic Dynamic Game

George, proudly wearing his *Game Club* hoodie, bursts into the dorm room one Thursday evening.

"Leo!" he shouts with the enthusiasm of a caffeinated squirrel. "I've signed up for the University Open House! The Game Club is getting a booth! It's going to be huge!"

Leo, halfway through restringing his guitar, blinks. "…booth?"

"Yeah," George grins. "You and the Music Club should join too! Imagine it — two best friends, side-by-side, ruling the event. Let's goooo!"

The Music Club, however, is still deciding whether to participate. If they do join, each club must choose **what to sell**: **food** or **drinks**.

Food is a bigger draw and yields a higher payoff.

Drinks are cheaper and easier to manage, but give a lower payoff.

If both clubs sell **the same item**, they will **split the same pool of customers.**

If they sell **different items, each gets their own crowd** and avoids competition.

But if only one club joins, whatever they sell will bring a big payoff — no competitors.

The Objective is clear: **Each club needs to figure out how to maximize their payoff, knowing the other club's decision affects their own success.**

The Open House countdown begins.

The tension rises.

And somewhere… Betty the cat is probably wondering why she's not invited 😾

GAME MODELING

The Open House Booth Battle: First 3 Days

The competition between George's Game Club and Leo's Music Club transforms into a **Dynamic Game**, a scenario where players move sequentially. Game Club is already signed up for the event, giving Leo's club the initial choice. *It is Leo and Music Club's turn to decide whether to join.* If they join, George's club picks their product first, then the Music Club chooses next.

<u>The Game Structure:</u>

We analyze this game using a **Game Tree** that plots the sequence of decisions, starting with Node 1 (N1), the Music Club's first move:

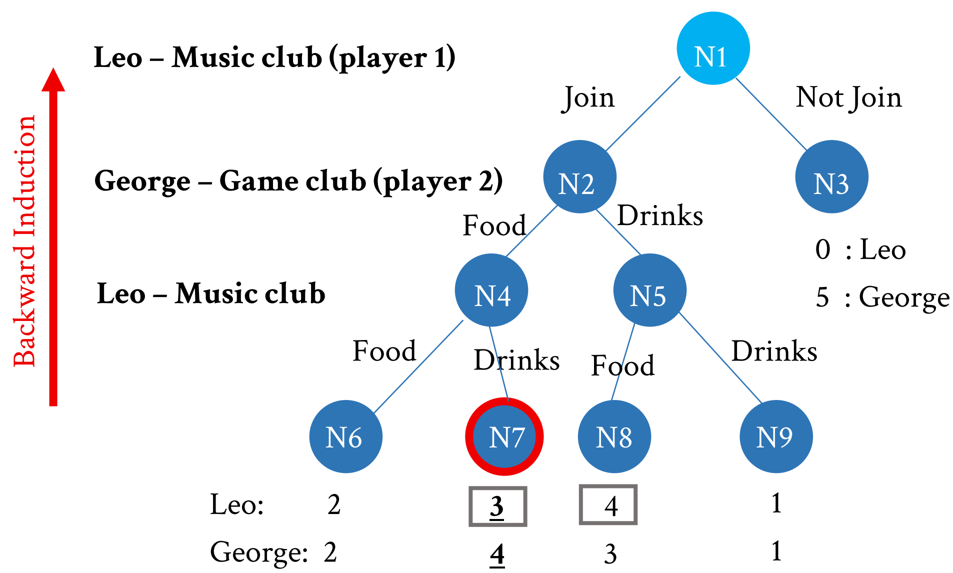


Figure 2.1.1 The Market Entry Game

Node 1 (Leo – Music club's move): Leo's club decides whether to **Join** the event or **Not Join**.

- Choosing **Not Join** leads immediately to the Terminal Node **N3** **(→ 0, 5)**. Leo's Music Club gets 0, and George's Game Club gets 5 for being the only club with a booth.
- Choosing **Join** leads to the second decision point, **N2**.

N2 (George – Game Club's Move): If the Music Club joins, George's club makes the first product choice: **Food** or **Drinks**, since his club signed up first.

- Choosing **Food** leads to **N4**.
- Choosing **Drinks** leads to **N5**.

N4 (Music Club's Response | George chose Food): If the Game Club committed to **Food**, Leo's Music Club must now choose its product, weighing competition versus avoidance.

- If Leo chooses **Food**, they both sell the same product and split the same pool of customers, leading to payoff of **(2, 2)** for (Leo, George) (Terminal Node **N6)**
- If Leo chooses **Drinks**, they avoid competition, but Food gives a higher return for George, leading to payoffs of **(3, 4)** for (Leo, George) (Terminal Node **N**7).

N5 (Music Club's Response | George chose Drinks): If the Game Club committed to **Drinks**, Leo's Music Club again faces the choice of competing or avoiding.

- If Leo chooses **Food**, they avoid competition, and in this pairing, Food gives a higher return for Leo, leading to payoffs of **(4, 3)** for (Leo, George) (Terminal Node **N8**).
- If Leo chooses **Drinks**, they both sell the same product and split the same pool of customers, leading to payoff of **(1, 1)** for (Leo, George) (Terminal Node **N9**).

<u>Solving with "Backward Induction"</u>

Since the game is dynamic, we solve for the unique, stable outcome using the concept of **Backward Induction**. This process starts by analyzing the last possible moves to determine the best strategy at every preceding node.

<u>Step 1: Starting from *the node above the terminal nodes*; N4, N5, to predict what the last player, Leo, will do</u>

- **At N4 (George chose Food):** Leo compares the payoffs for himself: choosing Food (**2**, 2) vs. choosing Drinks (**3**, 4). Leo's rational choice is **Drinks**, securing a payoff of 3 (→ **N**7).
- **At N5 (George chose Drinks):** Leo compares his payoffs: choosing Food (**4**, 3) vs. choosing Drinks (**1**, 1). Leo's rational choice is **Food**, securing a payoff of 4 (→ **N8**).

<u>Step 2: Moving *one more level up* to find what the earlier player, George, will do</u>

Now, we move back to George's move at N2, knowing exactly how Leo will respond to each of George's choices.

- If George chooses **Food** (N4), he knows Leo will respond with **Drinks** and the game ends at **N**7, giving him a payoff of **<u>4.</u>**
    - N2 → N4 → **N**7 (3, **4**).
- If George chooses **Drinks** (N5), he knows Leo will respond with **Food** and the game ends at **N8**, giving him a payoff of **3**.
    - N2 → N5 → **N8** (4, **3**).

George, as the rational player seeking to maximize his payoff, will choose Food, as 4 is greater than 3. This means the game path, if played, will lead to **N7.**

Step 3: Continuing Step 2 up the tree

Finally, we move to the very beginning of the game at N1 to see Leo's first move, knowing the complete outcome of the game if he chooses to join.

- If Leo chooses **Not Join** (N3), his final payoff is **0.**
    - N1 → **N3 (0**, 5).
- If Leo chooses **Join** (N2), he knows the game will progress through George's decision (Step 2) and lead to the outcome at **N7**, giving Leo a final payoff of **3.**
    - N1 → N2 → N4 → **N7** (**3**, 4).

Since a payoff of 3 is better than 0, Leo's initial rational choice is **Join**.

The only Nash Equilibrium of the game – Subgame Perfect Nash Equilibrium

The application of Backward Induction yields a single, stable outcome called the **Subgame Perfect Nash Equilibrium (SPNE).** This outcome is the only one that guarantees stability in every subgame, and it follows the path: N1 → N2 → N4 → N7

1. Leo: Chooses to "Join."
2. George: Chooses to sell "Food."
3. Leo's response: Chooses to sell "Drinks."

The final payoffs are **(3, 4)**, **Leo gets 3 and George gets 4.**

Leo is still considering *whether the Music Club should participate* in the open house.

This is not the first time the university has hosted this event, and last years' experience has taught them a lesson about this month-long event.

Here's the long-term twist: the open house starts with a bang, but **after the first three days, the hype for food booths completely disappears.** The high cost of ingredients turns selling food into a money-losing business. **Any club that sticks with food will end up with a negative payoff (-1).**

Drinks, on the other hand, are the reliable winners. They maintain their popularity throughout the entire event — quick, refreshing, and easy for students to grab on the go.

So, how will these new conditions change the game?

The payoffs have shifted dramatically, which means the players' best choices will likely be different. It is a new game entirely. The players have to recalculate their strategies based on this **long-term reality.**

The Open House Booth Battle: Long-Term Scenario

Just like before, George has already signed up, now Leo has to decide whether to join. If he does, George chooses what his club will sell first, then Leo chooses what his club will sell.

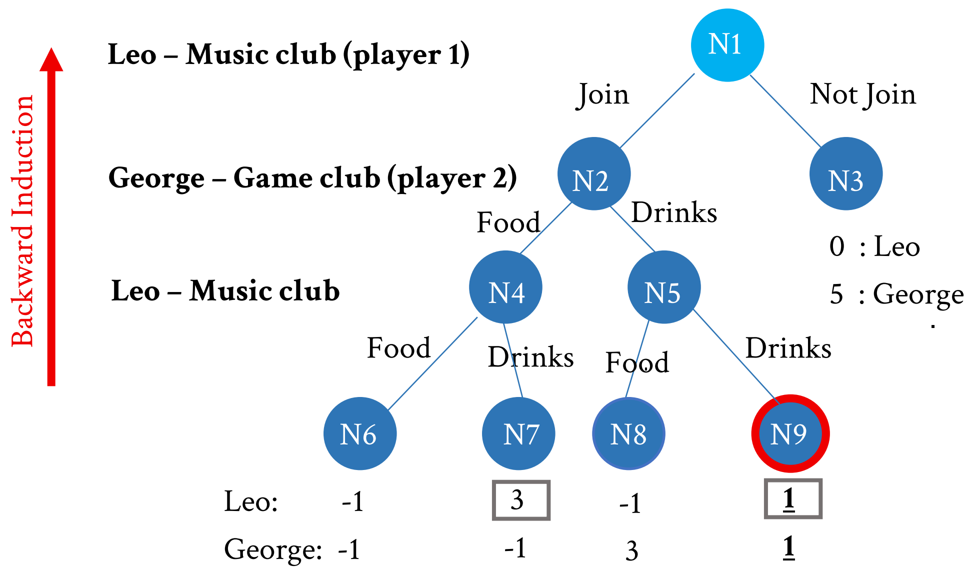


Figure 2.1.2 The Market Entry Game : Long-Term Scenario

<u>Solving with "Backward Induction"</u>

<u>Step 1: Starting from *the node above the terminal nodes*; N4, N5, to predict what the last player, Leo, will do</u>

- **At N4 (George chose Food):** Leo compares the payoffs for himself: choosing Food (**-1**, -1) vs. choosing Drinks (**3**, -1). Leo's rational choice is **Drinks**, securing a payoff of 3 (→ **N7**).
- **At N5 (George chose Drinks):** Leo compares his payoffs: Food (-**1**, 3)vs. Drinks (**1**, 1). Leo's rational choice is **Drinks**, with payoff of 1 (→ **N9**).

Step 2: Moving *one more level up* to find what the earlier player, George, will do

We move back to N2 and look at George's perspective, knowing how Leo will respond.

- If George chooses **Food** (N4), he knows Leo will respond with **Drinks**, giving George a payoff of -**1**.
    - N2 → N4 → **N7** (3, -**1**).
- If George chooses **Drinks** (N5), he knows Leo will respond with **Drinks**, giving George a payoff of **1.**
    - N2 → N5 → **N9** (1, **1**).

George, as the rational player seeking to maximize his payoff, will choose **Drinks**, as -1 is lower than 1. This means the game path, if played, will lead to N9.

Step 3: Continuing Step 2 up the tree

Finally, we move up to the very beginning of the game at N1 to see Leo's first move, knowing the complete outcome of the game if he chooses to join.

- If Leo chooses **Not Join** (N3), his payoff is **0**.
    - N1 → **N3 (0**, 5).
- If Leo chooses **Join** (N2), he knows the game will lead to the outcome at **N9**, giving Leo a final payoff of **1.**
    - N1 → N2 → N4 → **N9** (**1**, 1).

Since a payoff of 1 is better than 0, Leo will choose **Join**.

Subgame Perfect Nash Equilibrium (SPNE)

1. Leo: Chooses to "Join."
2. George: Chooses to sell "Drinks."
3. Leo's response: Chooses to sell "Drinks."

The **Subgame Perfect Nash Equilibrium (SPNE)** leads to the path N1 → N2 → N4 → **N9, with the final payoffs of (1, 1) where both get 1.**

Game Tree and Backward Induction

A **game tree**, or a game in extensive form, is a way to model scenarios where players make decisions sequentially. Dynamic games are easiest to explain in this form, while static games are typically shown in a game matrix. The game tree is written and read from the top down, but **we analyze it using a method called "Backward Induction"**, which works from the end of the game tree back to the beginning.

Subgame Perfect Nash Equilibrium (SPNE)

A **Subgame Perfect Nash Equilibrium** is a key concept for analyzing dynamic games, and it is found **using Backward Induction.**

To understand it, first think of a game tree as having smaller games inside of it, called "subgames". A subgame is just a smaller game that starts at any decision node in the larger game tree. An SPNE is **a set of strategies** (the players' choices) **that form a Nash Equilibrium in "every subgame."**

In other words, the SPNE ensures that every player's strategy is optimal not just from the beginning of the game, but from any point forward. By using Backward Induction, we find the best possible choice for each player at every single decision node. The strategy that results from this process is the subgame perfect equilibrium.

Dynamic Game's Components

Dynamic games have similar components to static games, but with a key difference in the ***Sequence***:

1. **Players:** The individuals or groups making choices in the game, like George (Gaming Club) and Leo (Music Club).
2. ***Sequence***: This is the most important difference from static games. Players move in a specific order, and each player's decision is influenced by the choices made before their turn.
3. **Actions:** The actions are the choices players can make at each turn of the game.

4. **Payoffs:** The payoffs represent the utility, or satisfaction, that each player gets from a final outcome. They are written as *(Payoff for Player 1, Payoff for Player 2)*. A higher number is always better. The payoffs are designed to reflect the scenario, such as winning is good, fighting is bad, and compromising is neutral.

GAME TRANSFORMATION

Short-Term vs. Long-Term

The two "Open House Booth Battle" scenarios show that *an equilibrium is not a fixed result*; it shifts based on the environment or game conditions. This principle applies to both static and dynamic games.

George's club and Leo's club notice that the short-term gain will turn into a negative payoff **dilemma** in the future due to the change of environment, so they shift to (Sell Drinks, Sell Drinks) to avoid that. As introduced in our second core approach to establish cooperation: **Modifying the Game's Structure, leading to the change in payoffs, can create a new "Win-Win" equilibrium**. We saw this previously in the Prisoner's Dilemma when Leo's noise-canceling headphones altered the payoffs and resolved the conflict.

In this episode, we apply this logic to explore **how a shift from a short-term to a long-term perspective reveals the underlying motivations of players** in real-world scenarios.

- **Short-Term Game:** In the initial game, high food payoffs in the first 3 days made it rational for clubs to sell different items. By avoiding direct competition, each club sought to maximize their individual immediate reward.
- **Long-Term Game:** However, when analyzing the game over a month-long period, market saturation or changing demand made food a losing option. This forced a new equilibrium where both clubs chose to sell drinks. Even though they had to compete and accept smaller profits, it was the only strategy that guaranteed a positive payoff over time.

Application

Real-World Example: Tech R&D and Market Exit

This shift from short-term to long-term payoffs explains why firms often make seemingly counter-intuitive decisions. For instance, a tech company might invest billions into a new market (like AI or green energy) that remains unprofitable for years — they are **ignoring short-term losses to secure a dominant position in a high-payoff long-term game**. Conversely, a firm might exit a currently profitable market if they anticipate future risks or a declining long-term payoff. Ultimately, the strategic choices of rational players are always dictated by the payoffs they expect over the **entire course** of the game, rather than a single snapshot in time.

EPISODE 2.2 - The LAST Bun

Hawk-Dove Game (Dynamic Game Version)

Earlier in the week, a kind neighbor gifted George and Leo a box of fifteen fluffy steamed buns… soft white dough, still warm when they got them, with fillings of sweet pork and a hint of sesame.

Over the past few days, they have been enjoying them one by one.

Now, only *one* remains.

It is 11:30 PM on a Sunday night. The local shops are closed, food delivery has stopped, and neither of them has the energy to walk to the 24-hour convenience store. This single bun is the last bite left in the apartment.

Both walk into the kitchen at the same time, eyes locked on the fridge.

Both know exactly what's inside.

Both want it.

The Objective: **Decide who gets the last bun.**

The choice and the outcome will set the roommate vibe for the rest of the night… and maybe into tomorrow.

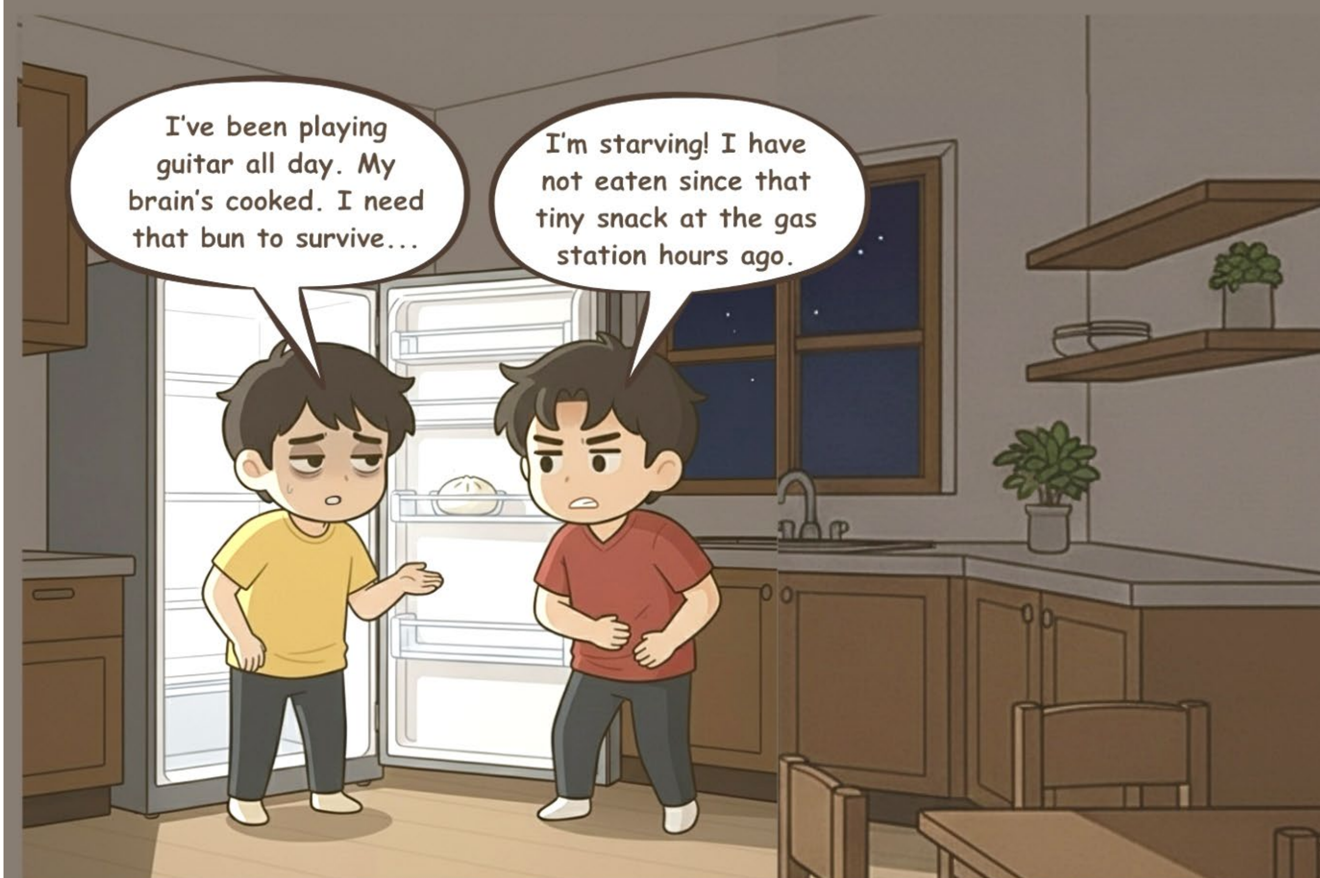
George has just returned from a weekend back in his hometown, where he went hiking with his family. He is relaxed, happy, but hungry after the long bus ride.

Leo, on the other hand, has spent the weekend cramming for his music club's small concert. Twelve hours of guitar practice. His fingers ache. His mind is fried. For days, he has been keeping himself going with one thought: that last bun.

"I've been playing guitar all day. My brain's cooked. I need that bun to survive," Leo says.

"I'm starving! I have not eaten since that tiny snack at the gas station hours ago," George replies.

They lock eyes. The fridge hums in the background. The unspoken question hangs heavy in the air —*Who makes the first move?*



GAME MODELING

Who makes the first move?

The conflict over a single, desirable item, the last bun, shifts George and Leo's interaction into a **Dynamic Game** that models the classic **Hawk-Dove Dilemma**. Assume *Leo*, who feels stressed the whole day in this scenario, decides to move first, forcing George to respond.

<u>The Game Structure:</u>

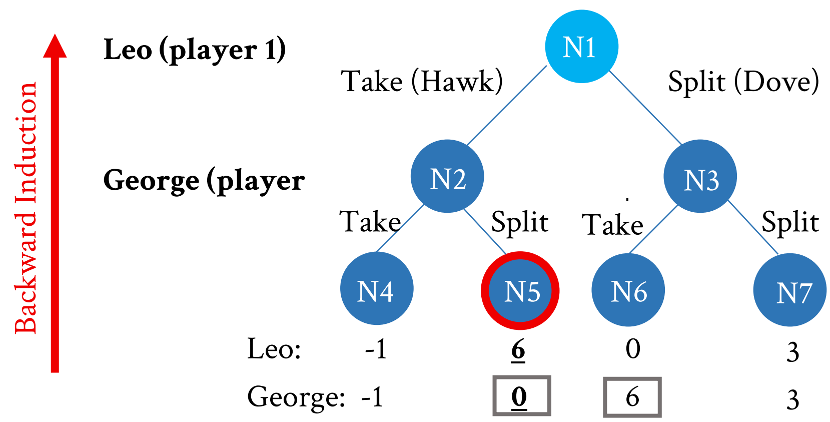


Figure 2.2.1 Tree of Hawk-Dove dynamic game

Node 1 (Leo's move): Leo's first action determines the structure of the rest of the game.

- Choosing **Take** (Hawk) leads to Node 2.
- Choosing **Offer Split** (Dove) leads to Node 3.

Node 2 (George's Response | Leo chose Take): If Leo aggressively takes the bun, George must react.

- If George chooses **Take** (fighting back), the result is an argument, and the bun sits uneaten. Both are unhappy and hungry, leading to the low mutual payoff of **(-1, -1)** for (Leo, George) (Terminal Node **N4**).
- If George chooses **Offer Split** (capitulation to avoid conflict), the aggressive first mover (Leo) secures the entire surplus, leading to the highly unequal payoff of **(6, 0)** for (Leo, George) (Terminal Node **N5**).

Node 3 (George's Response | Leo offered Split): If Leo attempts cooperation by offering a split, George must respond.

- If George chooses **Take** (exploiting the offer), George gains the entire surplus, and Leo is upset, leading to the highly unequal payoff of **(0, 6)** for (Leo, George) (Terminal Node **N6**).
- If George chooses **Offer Split** (reciprocating cooperation), they achieve the fair, moderate outcome for both, leading to payoffs of **(3, 3)** for (Leo, George) (Terminal Node **N7**).

Solving with "Backward Induction"

Step 1: Starting from *the node above the terminal nodes*; N2, N3, to predict what the last player, George, will do

- **At N2 (Leo chose Take):** George compares his payoffs: Taking (-1, **-1**) vs. Offering Split (6, **0**). George's rational choice is **Offer Split**, as 0 is better than the negative outcome -1 of a fight (→ **N5**).
- **At N3 (Leo offered Split):** George compares his payoffs: Taking (0, **6**) vs. Offering Split (3, **3**). George's rational choice is **Take**, as 6 is the highest possible payoff for him (→ **N6**).

<u>Step 2: Moving *one more level up* to find what the earlier player, Leo, will do</u>

Now, we move back to Leo's initial move at N1, knowing exactly how George will rationally respond to either choice.

- If Leo chooses Take (N2), he knows the game will end at **N5**, giving him a payoff of <u>**6.**</u>
    - N1 → N2 → **N5** (**6**, 0).
- If Leo chooses Offer Split (N3), he knows the game will end at **N6**, giving him a payoff of **0**.
    - N1 → N3 → **N6** (**0**, 6).

Since a payoff of 6 is significantly better than 0, Leo's initial rational choice is **Take**.

<u>**Subgame Perfect Nash Equilibrium**</u>

The final actions and payoffs are:

1. Leo: Chooses to "Take" (Hawk)
2. George's response: Chooses to "Offer Split" (Dove)

The game ends at the terminal node N1 → N2 → <u>**N5**</u>**, with the final payoffs of** <u>**(6, 0)**</u>**,** where Leo gets 6 and George gets 0.

Since Leo always chooses the option that gives him the highest payoff, when he moves first, he chooses "Take" (Hawk). This illustrates the power of being the first mover in a Hawk-Dove game: **Leo leverages his initial aggression, and George rationally capitulates to avoid the mutually destructive outcome of a fight.**

Similarity Between 'Pure Nash Equilibrium' and 'SPNE'

The outcomes of the Hawk-Dove game in both a static (Using Pure Strategy) and a dynamic setting are very similar, but the process is fundamentally different.

In a **static game**, there are two possible **Pure Nash equilibrium: (Hawk, Dove) and (Dove, Hawk).** The players move *simultaneously*, so neither can force a specific outcome. The game could end in either of these two scenarios, and players are left to hope they are not the one who ends up with the worse payoff.

However, in a **dynamic game**, the *sequence* of moves forces a single, predictable outcome. Using Backward Induction, we find a single **Subgame Perfect Nash Equilibrium (SPNE)**. This equilibrium is still a **one Hawk, one Dove** result, but **the player who moves first will "always" choose to be the Hawk, which then forces the second player to be the Dove** to avoid a mutually destructive conflict.

This difference highlights the concept of a **first-mover advantage**. In the dynamic version, the player who moves first gets the better payoff. This gives that player a powerful advantage that **does not exist in the static game.**

- **Static Game: Matrix, Simultaneous, Single/Multiple Nash Equilibrium**

		Leo	
		Take	Split
George	Take (Hawk)	(3, 3)	**(0, 6)**
	Split (Dove)	**(6, 0)**	(-1, -1)

Figure 2.2.2 Matrix of Hawk-Dove static game

- **Dynamic Game: Tree, Sequence, Subgame Perfect Nash Equilibrium (SPNE)**

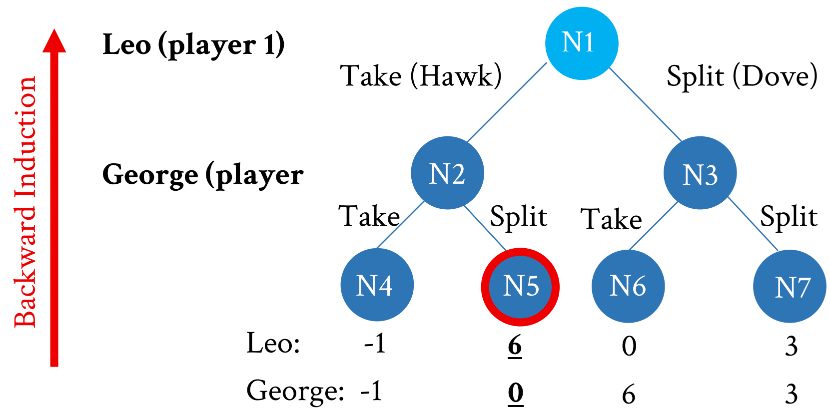


Figure 2.2.3 Tree of Hawk-Dove dynamic game

Everyone Wants to be Hawk

From what we learned in episode 1.3, the Static Hawk-Dove game is a situation where each player prefers to let the other bear the cost. Everyone would rather be the Hawk, but if there are two Hawks, it leads to a *dilemma*, "war," which is bad for both. This means it is crucial for **at least one player to act as a Dove**.

Because the Hawk's main goal is to take advantage of the Dove's willingness to avoid conflict, this creates a powerful **first-mover advantage** in the dynamic version of this game. **The player who moves first will choose to be the Hawk** and aggressively take the majority of the resource, this move **forces the second player to be the Dove to avoid a mutual dilemma**. For example, from the last bun scenario, Leo's "Hawk-like" behavior, Take, forces George to play the "Dove" role by choosing to Offer Split, resulting in an equilibrium where Leo gets 6 and George only gets 0. George makes this choice because 0 is better than the -1 he would receive from a mutual conflict.

In conclusion, **being the first to act gives a player the power to claim the best payoff, which makes starting the game a huge advantage**. However, we can only draw this conclusion in the Hawk-Dove game, because not every dynamic game results in a first-mover advantage. In some games, the observer will benefit more. For instance, in George and Leo's Open House Booth Battle (First 3 Days), Leo's Music Club started first, but the SPNE led to a final payoff of (3, 4). In this outcome, George, as the second player to move, claimed the slightly higher payoff of 4, while Leo received a payoff of 3.

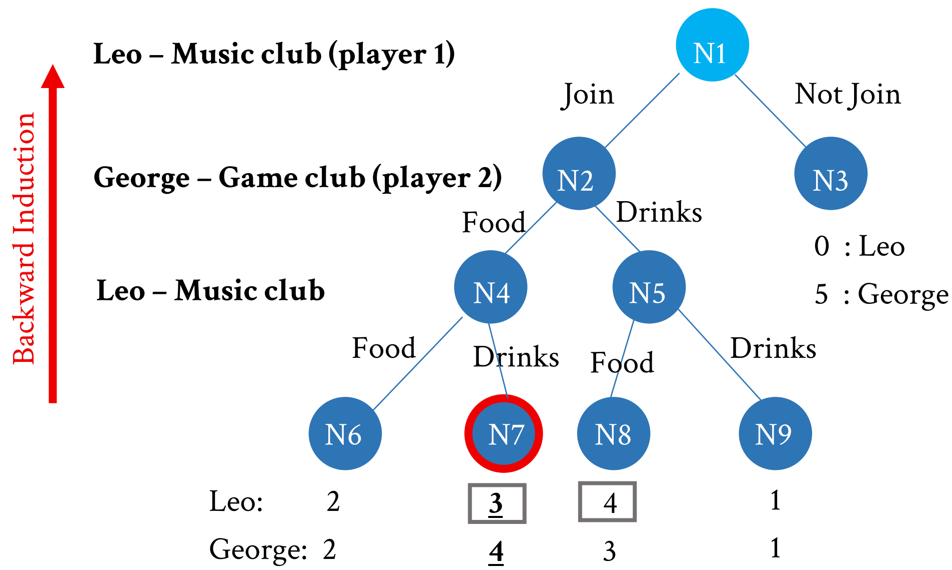


Figure 2.2.4 Observer's advantage case

GAME TRANSFORMATION

How to Reach the Win-Win outcome?

To solve a mutual dilemma, we could use the same method introduced in static version of Hawk-Dove game: **Establishing Cooperation by building trust through communication, agreements, or third-party support**.

George and Leo are roommates, not strangers. This matters.

Any aggressive move today will become **reputational debt** tomorrow.

George looks at the bun for a moment, then waives his hand.

"You take it," he says. "I've been eating too much this weekend anyway. Might as well start losing some weight."

Leo blinks. "You're serious?"

George nods. "Yeah. One bun isn't worth fighting over. I'll just sleep it off."

Leo hesitates, then slowly heats the bun.

The tension in the kitchen drains away as quickly as it came. There is no argument, no negotiation, no winner or loser.

The game never gets played. With communication and trust, one player simply refuses to compete. So, the problem solved and the *game disappears.*

Application

Real-World Example: First-Mover Advantage in the Tariff War

Although the game tree can clearly show who moves first, in real-life situations, it is often very hard to determine who the **first mover** is. However, the player with more power, influence, or the ability to act quickly is *often* the one who can claim this advantage.

For example, when President Trump was elected, his administration acted as the first mover. By announcing a **tariff war,** the US took on the role of the Hawk to aggressively pursue its goals of restricting China's economic influence and encouraging manufacturing to return to the US. These moves forced China to respond, putting them in the position of the second player who had to either retaliate: leading to a *dilemma of war*, or yield: playing the role of the Dove.

EPISODE 2.3 - The Hunger Game 🛵

Market with intermediaries

It was late at night in the rental apartment, but tonight the whole building smelled like *barbecue*. Somewhere down the hall, a party was sizzling.

George paused his game and looked at Leo.

Leo pulled off his headphones, opened the curtain… heavy rain poured outside.

"So… what do we do about our poor stomachs?" he sighed.

Just then, there was a knock on the door. George opened it, and the BBQ smell hit even stronger. His stomach growled.

At the door stood their neighbor… **Sara.** She was new to the building, friendly, and also a student at CTT University. Tonight, though, she wore the same look of hunger.

"Someone's barbecue is killing me," Sara groaned. "I just want to find someone to share the delivery fee with."

Now, three starving friends huddled over a delivery app, desperate for food. But it was late. The restaurants were almost sold out. Each had only one dish left.

Luckily, three restaurants were still open.

Since George, Leo, and Sara were too hungry to care about the food type, they treated them simply as Restaurant 1 (R1), Restaurant 2(R2), and Restaurant 3 (R3).

So, the hunger game begins: three buyers, three sellers, and only one dish each. *Who gets matched with whom?*

GAME MODELING

We are starving!

The game features three types of rational actors: **Buyers** (George, Leo, Sara), **Sellers** (three identical restaurants, R1, R2, R3), and **Intermediaries** (UberEats and Foodpanda).

- George, Leo and Sara are **indifferent to which restaurant or type of food they receive**, as long as they get a meal, they are satisfied.

The value structure is defined by the players' utility for a single meal:

- **George and Leo** each value a dish at **3**, and **Sara**, being the hungriest, values a dish at **5**.
- **Sellers value their last dish at 1**, assuming only cost, no profit, as they simply want to sell it quickly.

The network structure dictates trading partners and platform access, creating two distinct market conditions: **monopoly** and **perfect competition**.

- **Leo** only has **UberEats**, **Sara** only uses **Foodpanda**, while **George** use **both.**
- **R1** connects to **UberEats**, and **R3** connects to **Foodpanda** only, while **R2** connects to **both** platforms.

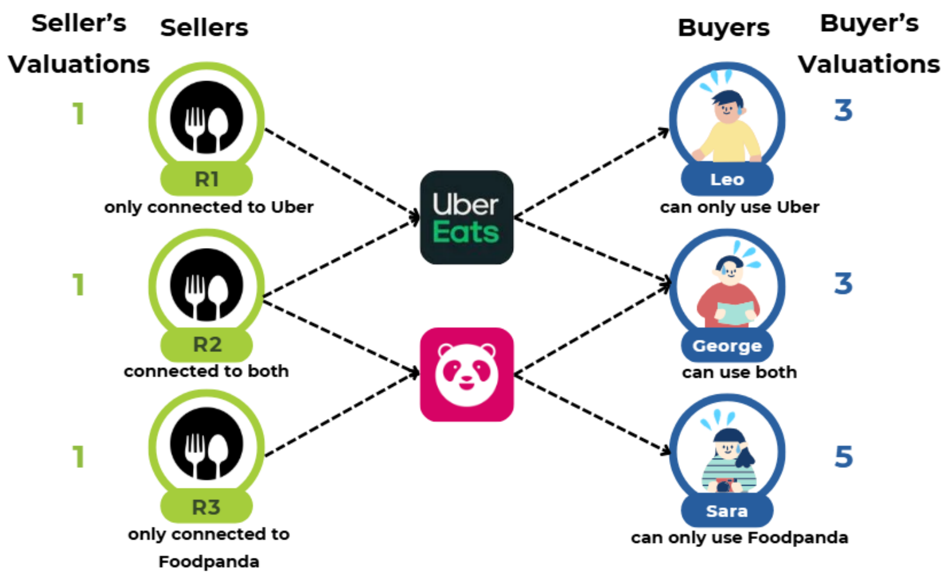


Figure 2.3.1 Trading Network

Monopoly Intermediation: Leo and Sara's Trades

The trading environment for Leo and Sara is characterized by intermediary monopolies, creating a highly unequal bargaining dynamic. For example, Restaurant R1 connects exclusively to UberEats, and Leo exclusively uses UberEats. This grants **UberEats monopoly power** over the R1-Leo trade. Similarly, **Foodpanda holds a monopoly** between Restaurant R3 and Sara.

1. **The Intermediary** (UberEats in Leo's case/FoodPanda in Sara's case) holding the most powerful bargaining power, **acts as the First-Mover** by setting non-negotiable prices that extract the entire economic surplus.
    - **The Leo & R1 Trade:** The total surplus between R1 and Leo is calculated as the Buyer's value minus the Seller's cost: 3 - 1 = 2. UberEats, as the monopolist, aims to capture the full surplus by setting the **ask price** (what Leo pays) at the maximum buyer valuation of 3 and the **bid price** (what R1 receives) at the minimum seller valuation of **1**.

- **The Sara & R3 Trade:** Due to Sara's higher valuation, the total surplus is larger: 5 - 1 = 4. Foodpanda sets the ask price at **5** and the bid price at **1** to capture the full surplus.

2. **Sellers and Buyers as Second-Movers:** Leo & R1, Sara & R3 have two choices: **accept** the price **or reject** it.

   - Leo's perspective: If he accepts, his payoff is 3 (his value) - 3 (ask) = 0.
     If he rejects, his payoff is 0. **Since he is indifferent but hungry, he will accept, getting a food with a payoff of 0.**
   - R1's perspective: If it accepts, its payoff is 1 (bid price) - 1 (cost) = 0. If it rejects, its payoff is 0. **Since R1 is indifferent but wants to sell the last dish, it will accept and sell the food with a payoff of 0.**
   - The outcomes are identical for the Sara and R3 case. Sara will accept ask of 5 and get food with a payoff of 0. R3 will accept bid of 1, selling food with a payoff of 0.

In both monopoly trades, since accepting yields a payoff of 0 and rejecting yields a payoff of 0 (no food/sale), both parties are indifferent but choose to accept the trade. While the intermediaries capture all the surplus.

<u>Outcome of Monopoly:</u>

- The intermediaries capture all of the surplus — UberEats earns a profit of 2, and Foodpanda earns a profit of 4.
- **Buyers and sellers earn a payoff of 0.**

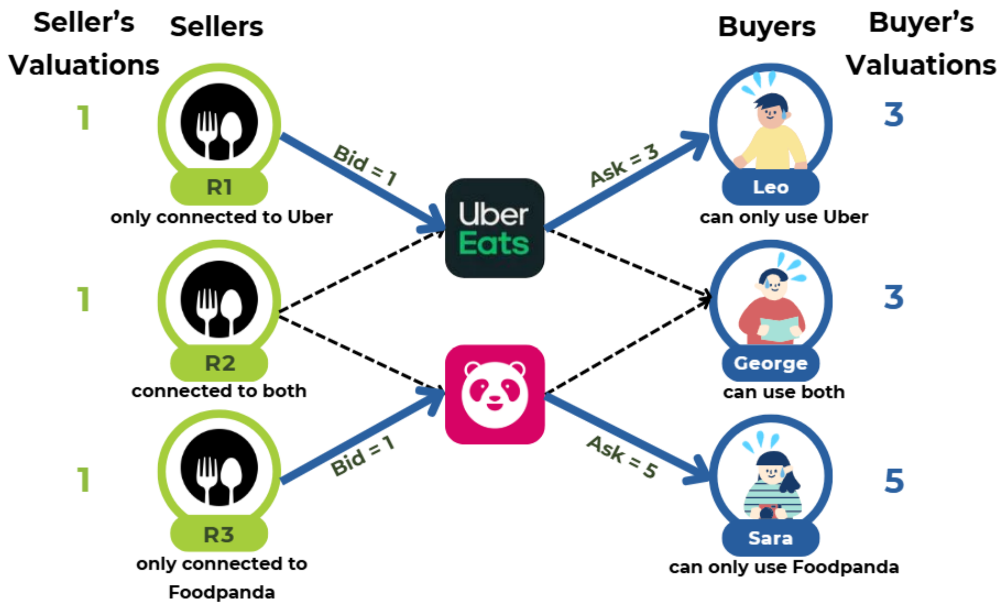


Figure 2.3.2 Intermediaries capture all of the surplus from Leo and Sara's trades

Perfect Competition Between Intermediaries: George's Trade

George and Restaurant R2 operate in a competitive market. George can use both UberEats and Foodpanda, and R2 also connects to both platforms. This overlap creates a situation of **perfect competition** between the two intermediaries for the R2-George trade.

1. **Both Intermediaries acts as simultaneous First-Mover:** each incentivized to win George's and R2's trade by making the best offer:

- As Intermediaries want to be chosen, they will **undercut** each other's ask price (to attract the buyer — starting from 3, 2.98, 2.80, …) and **increase** their bid price (to attract the seller — starting from 1, 1.9, 2.5, …).

This price competition continues until the bid and ask prices converge to a single **equilibrium price**, **bid = ask = x**, where the intermediaries can no longer make a profit, effectively earning **zero profit**. The total surplus between R2 and George is 2 (3 - 1 = 2). Since the intermediaries extract no value, **this surplus must be split entirely between the buyer and the seller**.

2. **Sellers and Buyers as Second-Movers:** R2 and George have two choices: **accept** the price **or reject** it.

   - The seller and buyer are **indifferent between the two intermediaries** because both intermediaries, through competition, **offer the same equilibrium price (x)**. So, let's assume R2 sells food to George through UberEats.

Outcome of perfectly competition:

- **Intermediaries earn a 0 profit.**
- The equilibrium price x, will fall in the range **1 < x < 3**. Any price in this range ensures both George (Buyer) and R2 (Seller) receive a positive payoff, which is a superior outcome to the 0-payoff seen in the monopoly case. For illustration, if x = 2, George's payoff is 1 (3 - 2), and R2's payoff is 1 (2 - 1).

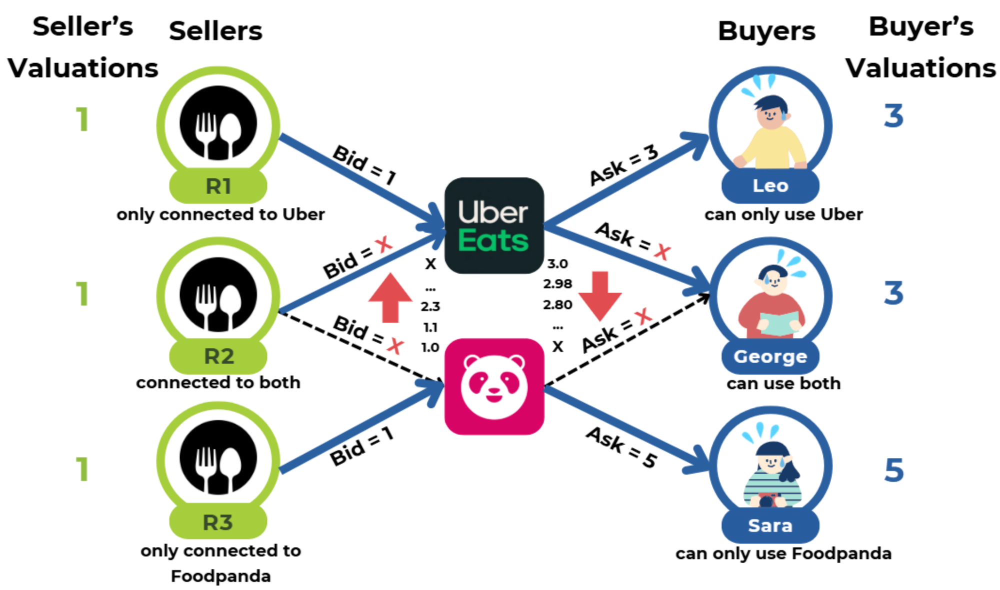


Figure 2.3.3 The equilibrium of perfect competition has a bid and ask of *x*

Possible Nash Equilibrium

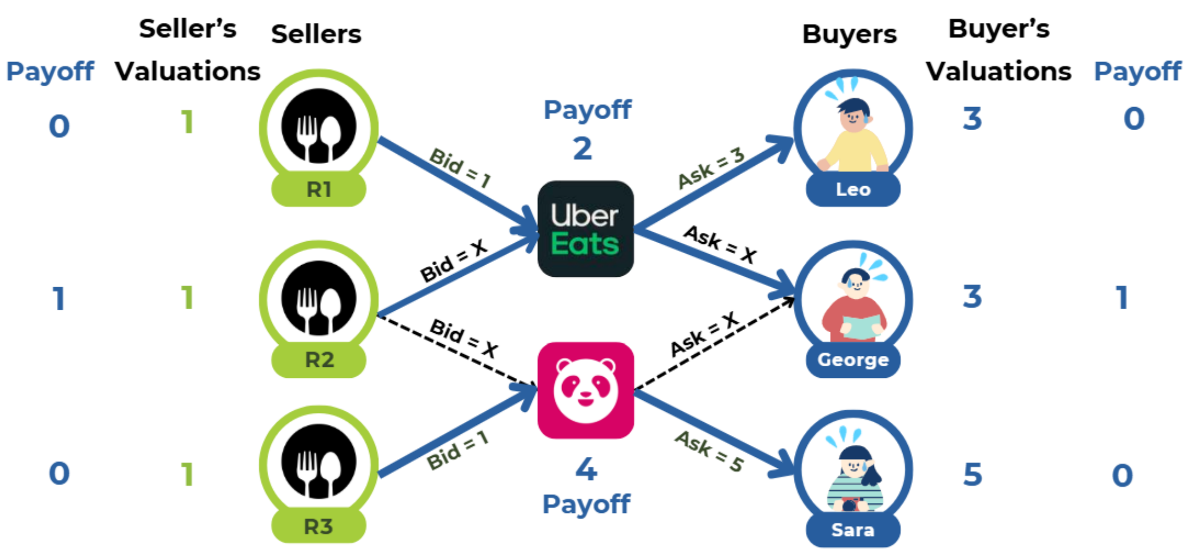


Figure 2.3.4 Trading Network at equilibrium

1. **Monopoly Trades (R1 & Leo and R3 & Sara):** The intermediaries use their power to capture the full surplus. The buyer and seller accept a payoff of 0.
   - The first dish flow from R1 through UberEats to Leo, with bid at 1 and ask at 3.
   - The second dish flow from R3 through FoodPanda to Sara, with bid at 1 and ask at 5.
2. **Competitive Trade (R2/George):** The intermediaries compete, resulting in the bid and ask prices converging to a single equilibrium price, x.
   - The last dish flow from R2 through any Intermediary (assume **UberEats**) to George, with bid = ask = **x**, where $1 < x < 3$. The price of x is not a single number but a continuum of values in the range between 1 and 3. Any of these can be the equilibrium price, but the result is always a split of the surplus between the buyer and seller.

Just as they were about to lock in their orders — Sara with Foodpanda, Leo with UberEats, and George through either app (likely UberEats) — the TV in the living room flickered on with an ad:

'Hungry? Don't limit your choices. ***Try Deliveroo or Foodomo!***

No downloads needed — just order straight from the website.'

The three froze.

Sara's eyes widened. "Wait… I don't have to clear storage on my phone anymore?"

Leo leaned forward. "Yes! Finally, I can keep my music apps and still order food."

George snapped his fingers, realization dawning. "Well… that changes everything, doesn't it?"

With Deliveroo and Foodomo on the table, now all three of them had equal access to every restaurant.

The late-night hunger game wasn't over yet.

Here Come Deliveroo and Foodomo!

The introduction of **Deliveroo** and **Foodomo** drastically alters the market structure, eliminating all previous intermediary monopolies and establishing a state of **Perfect Competition** across all trades. This shift significantly increases the bargaining power of the buyers (George, Leo, Sara) and sellers (R1, R2, R3).

The value structure remains constant: Sellers value a dish at **1**, Leo and George at **3**, and Sara at **5**. The key change is the increased connectivity:

- **Leo's Trade (R1):** Previously monopolized by UberEats, Leo and R1 now have access to **UberEats and Foodomo**.
- **Sara's Trade (R3):** Previously monopolized by Foodpanda, Sara and R3 now have access to **Foodpanda and Deliveroo**.
- **George's Trade (R2):** Remains a competitive trade between **UberEats and Foodpanda**.

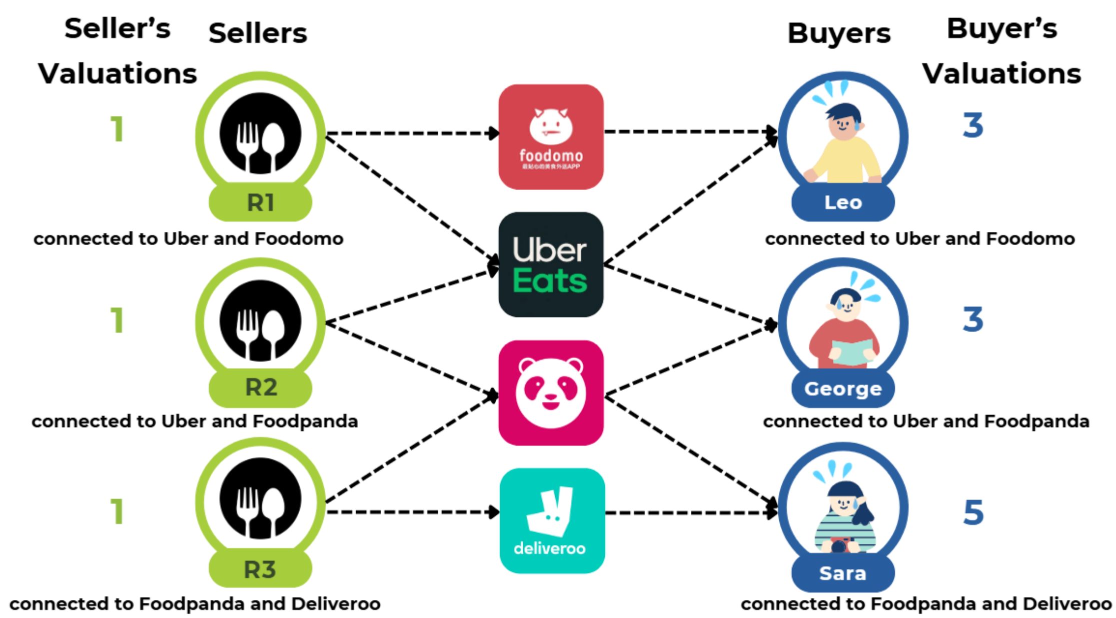


Figure 2.3.5 Trading Network after adding new intermediaries and links

The Rise of Perfect Competition Between Intermediaries

Since buyers and sellers are now connected to more than one intermediary, the market is characterized by intense price competition. This creates a situation of **perfect competition among the intermediaries.** For Leo, who previously relied on one intermediary, the market changes significantly. Now, UberEats and Foodomo must compete with each other for his and R1's trade. In all three trades (R1-Leo, R2-George, R3-Sara), the dynamics are identical:

1. **Intermediaries as First-Movers:** All platforms move first, competing aggressively to be the chosen intermediary for the trade.
    - To win the seller (R1, R2, or R3), intermediaries must **increase their bid price** (what the restaurant receives). To win the buyer (Leo, George, or Sara), they must **lower their ask price** (what buyer pays).

Assume the final equilibrium price of three trades (R1-Leo, R2-George, R3-Sara) are y, z, w, respectively. This competitive pressure continues until at the **bid price equals the ask price at a specific equilibrium**, driving the intermediaries' profit to **zero** in every transaction.

- Leo and R1 receive an offer at bid = ask = **y** , where 1 < y < 3.
- George and R2 receive an offer at bid = ask = **z**, where 1 < z < 3.
- Sara and R3 receive an offer at bid = ask = **w**, where 1 < w < 5.

2. **Sellers and Buyers as Second-Movers:** the sellers and buyers have two choices: **accept** the price **or reject** it.

   - The sellers and buyers are **indifferent** between the intermediaries because both, through competition, **offer the same equilibrium price.**

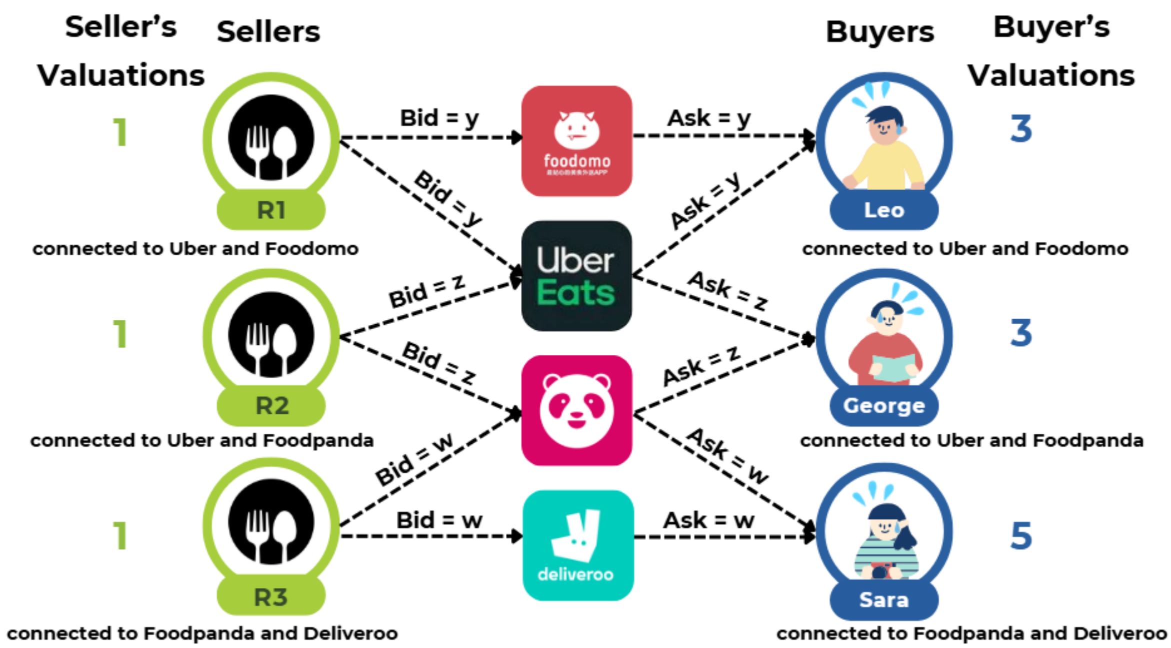


Figure 2.3.6 The equilibrium of perfect competition, where Intermediaries earn nothing

In this perfectly competitive environment, the entire economic surplus of the trade is guaranteed to be split between the buyer and the seller, while **intermediaries earn a 0 profit**:

- **R1 and Leo:** The total surplus is 2 (Leo's value 3 - R1's value 1). If we assume R1 sells food to Leo through **Foodomo** at an equilibrium price of **y = 1.5**, R1 gets a payoff of 0.5 (1.5 - 1), and Leo gets 1.5 (3 - 1.5).
- **R2 and George:** The total surplus is also 2. If we assume R2 sells food to George through **Foodpanda** at an equilibrium price of **z = 2**, R2 gets 1 (2 - 1), and George gets 1 (3 - 2).
- **R3 and Sara:** The total surplus is 4 (Sara's value 5 - R3's value 1). If we assume R3 sells food to Sara through **Deliveroo** at an equilibrium price of **w = 4**, R3 gets a payoff of 3 (4 - 1), and Sara gets 1 (5 - 4).

Possible Nash Equilibrium

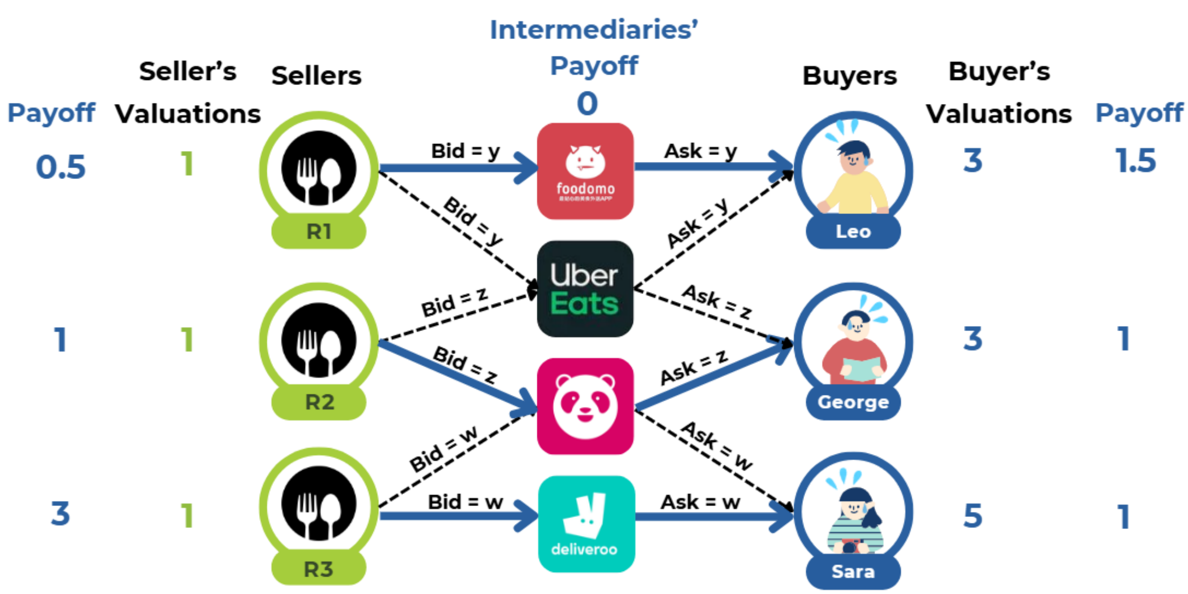


Figure 2.3.7 New Trading Network at equilibrium

1. The first dish flow from R1 through any intermediaries (assume **Foodomo**) to Leo, with bid = ask = y, where $1 < y < 3$.
2. The second dish flow from R2 through any intermediaries (assume **FoodPanda**) to George, with bid = ask = z, where $1 < z < 3$.
3. The last dish flow from R3 through any intermediaries (assume **Deliveroo**) to Sara, with bid = ask = w, where $1 < w < 5$.

This transition highlights the power of competition: the elimination of intermediary monopoly power, through the introduction of new platforms, ensures that **all buyers and sellers receive a positive payoff**, which is a significant welfare improvement compared to the monopoly cases where their payoff was 0.

What is Market?

The **market** is an economic system where sellers and buyers interact to exchange goods and services. Historically, geographic distance, trade barriers, or legal restrictions often prevented these seller and buyers from reaching each other directly. To bridge these gaps, **intermediaries** emerged to connect seller and buyer nodes, creating a network that facilitates transactions. These connections can be physical, such as the Panama Canal, or service-based, such as modern logistics and delivery networks.

In the context of game theory, every participant — whether a seller, buyer, or intermediary — is considered a "player." Each occupies a specific role with the primary objective of maximizing their individual payoff within the network.

Dynamic Games with Multiple Movers

Market games are modeled as **dynamic games**, where players move in a specific sequence. However, unlike simple sequential games where only one person moves at a time, each "turn" in a market game can involve **multiple players moving at the same time**.

The role of a player does not always determine when they move (e.g. being a seller does not automatically make you the first-mover). Instead, the player or *players* who have the ability to set the initial price or make the first offer are considered the **first-movers**.

- **Monopoly Market:** In a monopoly (such as the original Leo and R1 case), the single intermediary acts as the first-mover. They use this position to set the price and capture the entire surplus. The buyer and the seller then act as second-movers, simultaneously deciding whether to accept or reject the offer. Similarly, if a single seller holds a monopoly on a resource, they become the first-mover, setting prices for competing intermediaries.
- Perfect Competitive Market: In a competitive environment, multiple intermediaries act as **first-movers**, competing to make the best offer. This competition forces them to drive their prices to an equilibrium point where they earn zero profit, even though they move first.

The participants who respond to these offers are the **second-movers**. This structure shows how the game's outcome depends on the number of players in each role, and not just on who has the first turn.

A key assumption of this market game is **complete information**, which means every player knows their own valuation as well as the valuations of all other players. This is different from games with **asymmetric information**, where players have private knowledge, a concept we will explore later in Season 3 and Season 4.

Market Status: Monopoly Market

A player obtains **monopoly power** becoming the sole connection between two or more players in the network, creating an **essential edge.** This unique position allows the monopolist to act as the **first-mover**, setting prices to capture the **entire surplus** from a trade. Because the monopolist holds all the bargaining power, other players are often presented with an "all-or-nothing" offer; their only choice is to accept a payoff of zero and participate in the trade, or walk away with nothing.

Monopoly is often the earliest stage of a market, where buyers and sellers cannot contact each other directly. **The first intermediary to invest in a channel** such as a trade route or platform, **becomes the monopoly intermediary**.

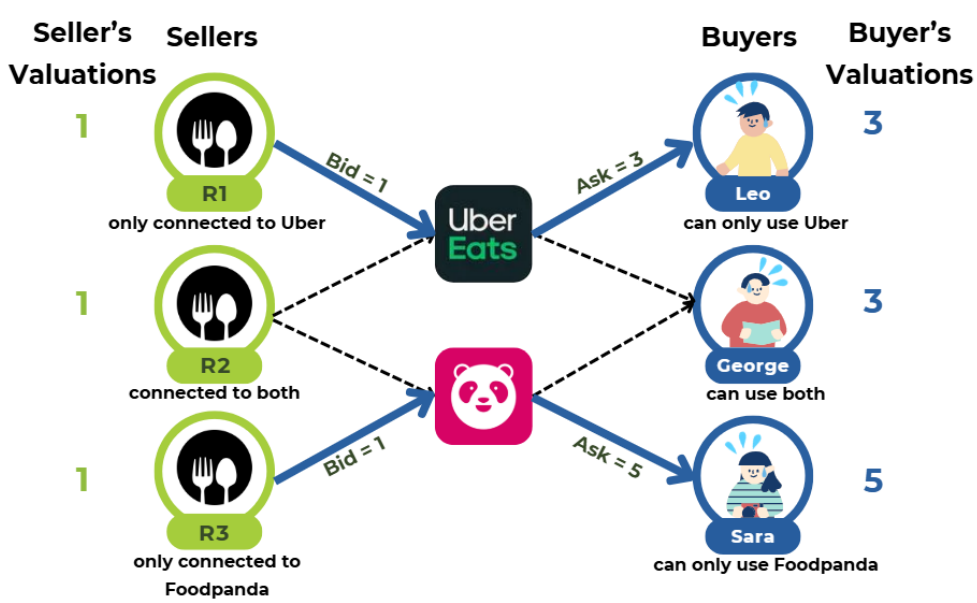


Figure 2.3.8 Monopoly Intermediaries in the cases of Leo and Sara, where intermediaries' control over the connection allowed them to dictate terms.

This monopoly power is not limited to intermediaries. A seller or buyer can also obtain this power by being the only party of their kind in a particular trading network.

- **One Seller vs. Two Intermediaries:** The single seller holds monopoly power as the exclusive source. Intermediaries will compete for the good, driving the bid price up to the buyer's maximum willingness to pay (buyer's value). The intermediaries will not get any profit as they get the item at the same price they sell to the buyer. The seller captures all the surplus, while the intermediaries and buyers earn zero profit.

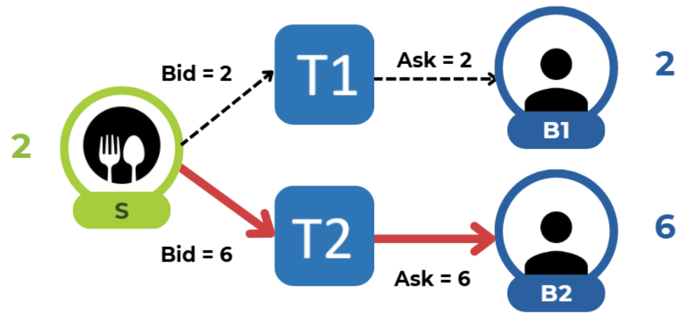


Figure 2.3.9 Monopoly Seller

- **One Buyer vs. Two Intermediaries and Two Sellers with Two Goods:** The single buyer holds monopoly power. Intermediaries compete to sell the good, driving the ask price down to their minimum cost (seller's value). The buyer captures all the surplus, while the intermediaries and sellers earn zero profit.

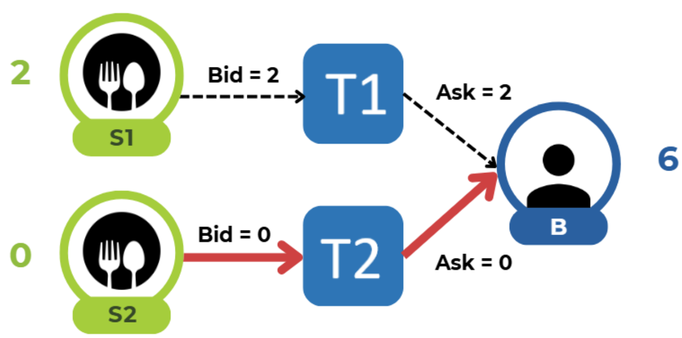


Figure 2.3.10 Monopoly Buyer

However, a monopoly is often a theoretical or temporary case. In a real-life market with no **barriers to entry**, when a monopoly exists and generates a high surplus, it creates an incentive for new players to enter the market to get a share. **This entry leads the market toward a perfectly competitive state**, which is more stable and efficient.

Market Status: Perfectly Competitive Market

The contrast between two version of our market game highlights the fundamental difference between monopoly and perfect competition regarding the distribution of wealth and efficiency. In **intermediaries monopoly** cases, intermediaries capture all the surplus, leaving buyers and sellers with nothing. Conversely, in **perfect competition** case, the competition among intermediaries forces them to offer prices that result in a zero profit. The total surplus is then split between the buyers and sellers, which is a much more beneficial outcome for the participants on the rim. This result represents the **market equilibrium**, which is considered **economically efficient**, as it leads to the best allocation of goods, maximizing the total welfare created in the market, the total sum of all payoffs in the system.

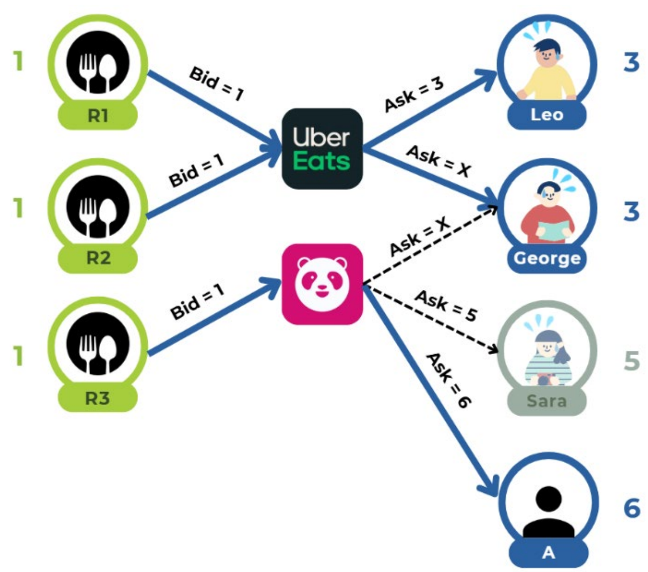


Figure 2.3.11 The flow of items is controlled solely by a single intermediary

Imagine we increase the number of buyers, but remove the connection between **R2** and **Foodpanda**. Now, all sellers are trapped under a monopoly structure.

The Restricted Network Outcome: At the Nash Equilibrium, the total payoff or **Social Welfare** of this monopoly case is: 6+3+3=**9**.

Then, we **add the edge** connecting **R2** to **Foodpanda**. Seller R2 is no longer restricted; R2 can now benefit from the competition between **Uber** and **Foodpanda**.

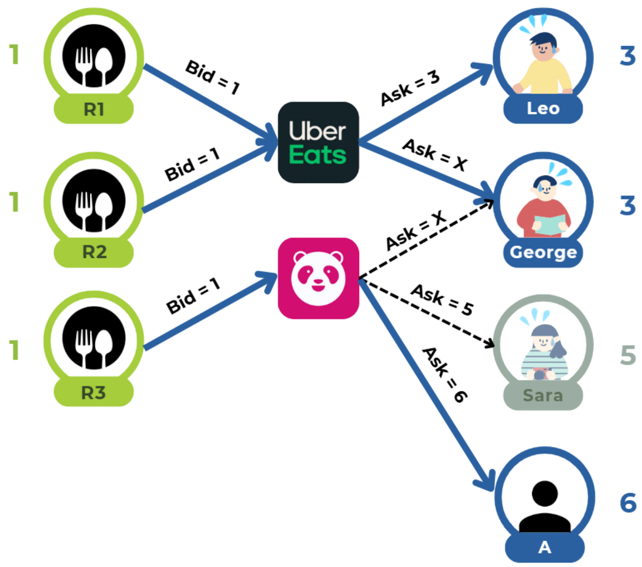


Figure 2.3.12 Equilibrium before vs. after adding a new link

The Restricted Network Outcome:

Social Welfare: 6+3+3=**9**.

The Competitive Expansion Outcome: Because the market mechanism allows items to flow more freely to the buyers who value them the highest, the equilibrium shifts. The new Social Welfare increases to: 3+5+6=**14**

In conclusion, this comparison proves that a competitive market is superior not just because it is "fairer" to buyers and sellers, but because it is more **efficient**. By adding just one connection (an edge in our network), the total value generated by the market jumped from **9 to 14**. Competition ensures that resources are allocated to their most highly valued use, maximizing the benefit for society as a whole.

GAME TRANSFORMATION

How to Reach the Win-Win-Win outcome?

As now we move to a **three-player** model involving the **Seller**, **Intermediary**, and **Buyer**, the game becomes more complex.

What is Win-Win-Win?

In this context, a "Win-Win-Win" refers to an improvement in **Social Welfare** — the total value created by the network — rather than just a simple Pareto improvement like in the Prisoner's dilemma episode.

We have established that social welfare is maximized in a **Perfectly Competitive Market** because goods flow freely to the buyer with the highest valuation. Therefore, we can say that the Nash Equilibrium in a Perfectly Competitive Market is a Win-Win-Win outcome. However, a deeper analysis of Win-Win-win outcome reveals a conflict of interest regarding the distribution of this value:

The Intermediary's Dilemma: In a perfect competition market, pressure drives the intermediary's profit to zero. While this is a "Win-Win" for the seller and buyer, it is a "Lose" for the intermediary.

The Seller and Buyer's Dilemma: In a monopoly market, the intermediary captures the entire surplus. While this is a "Win" for the intermediary, it is a "Lose-Lose" for the seller and buyer.

To find the true "Win-Win-Win," we must look at the evolution of the market:

1. **Stage 1 (No Market, No Trade):** Sellers and buyers are isolated. Total welfare is 0.
2. **Stage 2 (Emergence of the intermediary):** An intermediary invests in a *channel* (a bridge, a river route, or an app). This actually can call "Win-Win-Win" because **trade is better than no trade**. All parties benefit from the existence of a market that did not exist before.

3. **Stage 3 (The Monopoly Trap):** Once the channel is established, the intermediary accumulates monopoly power and uses it to capture all surplus. Although we could say the monopoly market helps seller and buyer improve from the no-market stage and know each other, this creates a dilemma of unfairness for seller and buyer.

4. **Stage 4 (Perfect Competition):** New intermediaries enter, breaking the monopoly. Social welfare improves to its maximum, achieving a "Win-Win-Win" equilibrium *in theory*, but this actually is intermediary's dilemma, a "Win-Win-Lose."

The Nash Bargaining Solution

If a monopoly is a "Lose-Lose-Win" (for the rim) and perfect competition is a "Win-Win-Lose" (for the middle), what is the outcome that provides fairness for all three sides?

The answer lies in the **Nash Bargaining Solution** (discussed in detail in episode 5.4). Once the players are connected and aware of each other after experiencing the frustrations of both dilemmas, communication and negotiation become the primary tools for stability.

By utilizing the **bargaining process**, a perfectly competitive market can transform into a true "Win-Win-Win" for all participants. In this stage, players negotiate a distribution of the surplus that acknowledges the value each party brings to the network. Their profit is based on bargaining result.

In a stable, **long-term market**, players may move away from the extreme of perfect competition, where intermediary gains zero, toward a **"Win-Win-Win" equilibrium** through bargaining. For example, a negotiated split in a competitive market might award a payoff of 1 to the Seller, 1 to the Intermediary (T2), and 2 to the Buyer (B2).

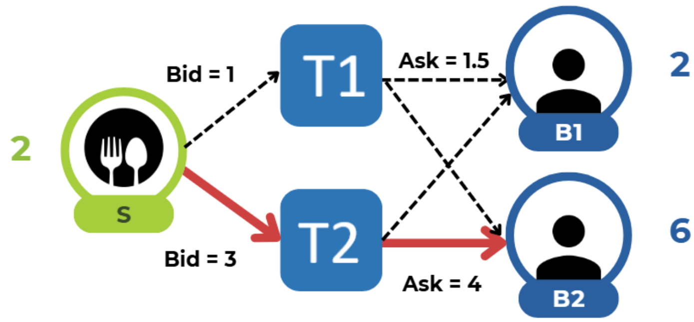


Figure 2.3.13 Equilibrium where Seller and Buyer negotiated a split

By shifting from aggressive price-taking to a **bargaining process**, the market achieves a "Win-Win-Win" where the **intermediary is rewarded for maintaining the channel, while the seller and buyer retain a fair share of the surplus they created.**

Application

Real-World Example: US and China Trade War

The US and China trade war illustrates the strategic use of market networks. A core objective of the Trump administration was to sever existing connections between the US's trading partners and China. By imposing tariffs and restrictions, the US sought to create a monopoly for itself on specific trading networks. This strategy aimed to force smaller countries to rely solely on US for certain goods and trade routes, thereby increasing the US's bargaining power to gain a larger share of surplus. This example shows how, in a network model, more choices (edges) are always better for nodes on the rim (small countries), as it increases their bargaining power, ensuring a more equitable share of surplus.

EPISODE 2.4 - Lucky You! 🤞

Repeated Reciprocal Game (Advanced)

Since last time George went hiking with his family when he returned to his hometown, this time he wanted to go with his best bro, Leo, to a far-away beach.

After checking in at the hotel and leaving their luggage, they walked along the shore and noticed a sign: **Under the Sea Adventure.**

"Hey bro, interested?" George nudged Leo.

"Um… but I've never snorkeled before," Leo said.

"Me neither! Let's try!"

A sailor, who had been waiting for customers, spotted them and waved. "Hey boys, *you don't look local. Traveling?"* He hid a sly smile, but George and Leo didn't notice.

"Oh, Uncle has a sharp eye! We came from another city. We wanna snorkel," George replied eagerly.

"Perfect timing! My customer just canceled because of heatstroke. I can take you to the most beautiful coral reef." He lowered his voice, as if sharing a secret. "Normally, you need to book one week in advance… lucky you! Only $4000."

Leo's eyes widened in shock. "…That's expensive."

"Okay, since you are students, I'll give you a discount — $3800?" Seeing their hesitation, he waved them toward the boat. "Alright, $3500. Cheapest price."

George and Leo couldn't reject him. They paid before the trip and got on his boat.

The uncle took them out to deeper water and handed them old snorkeling gear. George was the first to go down, but saw nothing. The water was too deep to spot any coral. The sailor only shrugged, "This season we can't go near the reef. Preservation rules."

Disappointed, George and Leo returned to the hotel. The receptionist told them the truth: *the trip was overpriced, and it wasn't even snorkeling season.* The same scam had happened to other travelers before, so there was already a warning posted on the wall in front of the hotel… but George and Leo hadn't noticed.

GAME MODELING

Why Does the Airport Taxi Cheat You?

The answer is the same as the question: Why did this boat sailor do this to George and Leo?

It is because he thought he could only meet George and Leo **one time.**

George and Leo are travelers. They might come to this beach only once in their lives, so the sailor and they will likely never see each other again. **The sailor has no established trust or relationship with George and Leo**, so his only focus is on how to get the most money from them in a single time.

But **if George and Leo were locals**, the sailor would be afraid to cheat them because they would likely meet him again, and the same scenario could be repeated. **He would likely cooperate** by being honest and tell them that it is not snorkeling season. **By doing so, the next time they want to snorkel, they will consider renting his boat.** And after their interactions and communication during the trip create trust, they might become his regular customers.

One-time Game vs. Repeated Game

What is the difference between a game played only once and a game that is repeated?

In a one-time game, this cooperation is nearly impossible. The game happens in a short period, and players who don't know each other have no trust. For example, in the beginning of the episode 1.1, George and Leo are new to each other and did not cooperate in the Dilemma Game. Moreover, since players will likely never see each other again, they have no reason to try communicating and building that trust. For these reasons, players will not cooperate. Their only goal is to get the highest possible payoff in that single moment. This is true for both static and dynamic games.

In the last season, we saw that some static games, like the **Prisoner's Dilemma** and the **Hawk-Dove Game**, require players to cooperate to achieve a better or fair outcome. In dynamic game version of Hawk-

Dove, the same idea applies. Players must cooperate by taking turns being the first mover to ensure a fair benefit for both. While we have discussed how communication can help with cooperation, it is actually **the ability to repeat the game that truly guarantees cooperation**, since trust is built over time.

<u>The Prisoner's Dilemma: From Conflict to Cooperation</u>

Let's use the classic **Prisoner's Dilemma** as an example. Player A and B are both suspects in a robbery and are caught at the same time. They don't know each other beforehand; they were just unlucky enough to be robbing the same bank. They are locked in separate investigation rooms to choose between staying silent or confessing.

	Win-Win	Player B	
		Stay Silent	Confess
Player A	Stay Silent	*(-1, -1)*	(-10, 0)
	Confess	(0, -10)	**(-4, -4)**

Dilemma

Figure 2.4.1 Silent or Confess?

If this is a one-time game, both players will choose to (Confess, Confess). Neither will choose to stay silent because they cannot trust the other to do the same. This non-cooperative choice results in the dilemma of both players confessing, leading to the outcome that hurts everyone (-4,-4).

What happens if players play this dilemma more than once? The game completely *changes.*

- **The First Time:** Both A and B will choose to (Confess, Confess). They do not know each other, so there is no trust.
- **The Second Time:** Both A and B will *still* likely choose to (Confess, Confess). Even with yesterday's eye contact on the walkway as police moved them between the investigation room and the cage, trust is not yet established.

- **A Few Times Later:** After this happens several times, they might start to wonder if they could both cooperate by choosing "Stay Silent." A silent look or a nod that they exchange on the walkway every day **might be enough to begin building a small amount of trust**.
- **Eventually:** After the game has been repeated *long enough*, a **"Win-Win" or Pareto outcome will finally happen**.

The key idea of a repeated reciprocal game is the **time period**. When players know there is a "next time," they will start to consider cooperation. But *how long does a game need to be played to ensure cooperation?* The answer is "**like infinity.**" If Player A and B play the Prisoner's Dilemma an infinite number of times, the result is guaranteed to be cooperation (Silent, Silent), as it is the best strategy for both.

Infinitely Reciprocal Game – The Math Behind Cooperation

In an infinitely reciprocal game, cooperation can be enforced by a **trigger strategy**. Think of it as a simple rule: "I will be nice to you, but the moment you betray me, I will be mean forever." This threat of permanent punishment can make cooperation a stable outcome.

This trigger strategy is a Nash Equilibrium if the players are patient enough to not be tempted by a short-term benefit. This holds true if the total value of a lifetime of cooperation is greater than the total value of a single act of betrayal followed by a lifetime of punishment. The player has no incentive to betray because the future loss is too great.

The more a player values the future, the more effective this threat is. These players are likely to be more patient than those who do not value the future. This "patience" is measured by the **discount factor (δ)**. The closer δ is to 1, the more patient the player, and the more likely the trigger strategy is to be a stable Nash Equilibrium. This means they value future cooperative payoffs more than the short-term gain from a non-cooperative choice. If this condition holds, the threat of infinite punishment is credible, and cooperation can be sustained.

Let's look at the payoff for the Prisoner's Dilemma to see why this works:

		Player B	
		Stay Silent	Confess
Player A	Stay Silent	*(-1, -1)*	(-10, **0**)
	Confess	(**0**, -10)	**(-4, -4)**

Figure 2.4.2 Playing reciprocal game could lead to Win-Win outcome

To find whether they will cooperate or not, we have to compare the present value of **Payoff of deviating** to the present value of **Payoff of cooperating**.

The formula for the present value is calculated using **δ**, the discount factor. This variable, ranging from 0 to 1, reflects how much a player values future payoffs compared to present ones.

- **Value or Payoff of deviating:** The one-time payoff from confessing (**0**) plus the discounted sum of all future "punishment" payoffs (-**4**) from the non-cooperative Nash Equilibrium.

If any player (assume B) deviates to Confess which gives higher payoff of **0** while Player A is playing Silent and taking full blame (-10). This will trigger A to Confess in the next stage, and they will not cooperate, *staying (Confess, Confess) forever after*.

$$\boldsymbol{V_d} = \underline{\mathbf{0}} + \delta * (-\mathbf{4}) + \delta^2 * (-\mathbf{4}) + \delta^3 * (-\mathbf{4}) + \ldots + \mathrm{L} = \frac{-\mathbf{4}\delta}{\mathbf{1}-\delta}$$

$\delta * (-\mathbf{4}) + \delta^2 * (-\mathbf{4}) + \delta^3 * (-\mathbf{4}) + \ldots$: the discounted stream of payoffs that Player B will receive in every subsequent period after his first betrayal stage. In this example, the payoff from the "punishment" phase is $(-\mathbf{4})$ in every period, discounted by δ. The formula for the sum of an infinite geometric series is: $\frac{a}{1-r}$ (a is the first term of the series, which is $-\mathbf{4}$δ. While r is the discount factor, δ.) So, the simplified formula is $\frac{-\mathbf{4}\delta}{\mathbf{1}-\delta}$

L : represents the discounted sum of these future payoffs $(-\mathbf{4})$.

- **Payoff of cooperating:** If no player deviates to Confess, both will continue silent. V_c is the sum of all future "stay silent" payoffs (*-1*), discounted by δ.

$$\boldsymbol{V_c} = \frac{-1}{1-\delta}$$

- **To make cooperation be the better choice,** the payoff of cooperating must be greater than or equal to the payoff of deviating: $\boldsymbol{V_c} \geq \boldsymbol{V_d}$

$$\frac{-1}{1-\delta} \geq \underline{\boldsymbol{0}} + \frac{-4\delta}{1-\delta}$$

$$\frac{-1}{-4} \leq \delta \text{ ; flip the inequality sign when dividing by a negative number}$$

$$\boxed{\frac{1}{4} \leq \delta}$$

By simplifying this inequality, we find that the condition for cooperation is: $\boxed{\delta \geq 0.25}$. This means *if players value the future by at least 25%, the threat of infinite punishment is credible enough to enforce cooperation.*

GAME TRANSFORMATION

How to Reach the Win-Win outcome?

As we have established, the "Win-Win", or Pareto optimal outcome is the most desirable but often hard to achieve in a single-interaction game. But when a game is played repeatedly, players start to value the future and expect a "tomorrow." They realize that the **short-term temptation to betray is outweighed by the long-term benefit of sustained cooperation**. This shifts the strategic calculation, making the Win-Win outcome of (-1, -1) the rational choice.

The Value of the Future – The Discount Factor (δ)

The concept of the infinitely reciprocal game helps us answer a philosophical question: *Why do people fight or start wars?*

The answer is not about the exact numerical payoffs those people would receive, but about how much a player **values** those payoffs. **It depends on one crucial factor: their perception of the game's duration**. Do they believe they are playing a one-shot game with no future consequences, or a long-term, infinitely reciprocal game?

If people or nations believe they only have **one life**, or this is the only chance to play, their actions are **tempted by the short-term benefit** of an aggressive, non-cooperative option. They ignore the long-term consequences, which **leads to a mutual loss** like the (-4, -4) dilemma — or even *war.*

However, if people perceive their interactions as an **infinitely reciprocal game**, the incentives shift dramatically. They become **interested in a long-term, sustainable benefit**. For example, some religions create a belief that bad actions will lead to a bad "next life." This encourages people to think they are in an infinite game, causing them to value the future. This value is measured by the **discount factor (δ)**, which reflects how much a player cares about future payoffs compared to present ones. A patient player with a high discount factor would be less likely to start a war because they value a sustained, cooperative relationship, or are hoping for a better new life.

Your analogy about the $1 million is a perfect example of this. Most people would prefer $1 million today because we live for a limited time and have a low discount factor for our own lives. This low discount factor is largely due to present bias, an evolutionary survival instinct that prioritizes immediate rewards over uncertain future ones, **causing us to value immediate utility more than future utility.**

Conversely, if we were immortal, $1 million today and $1 million next month would have the same value. They would have no reason to risk a conflict for an immediate payoff because they value the perpetual future more. This is the logic of an infinite game.

In conclusion, the crucial factor determining whether people cooperate is their **perception of time**. The longer the game, the more players **value the future** and become more patient. This makes them far more likely to see that mutual cooperation is a superior long-term strategy for everyone.

Application

How Repeated Reciprocal Games Change Human Behavior

In reality, we play repeated games far more often than one-time games. A very common example is teaching a child. Repeated reciprocal interactions can change human behavior, and even animal behavior. By consistently rewarding a child for good behavior and applying small consequences for negative behavior, the child learns what actions are beneficial and what actions are not.

Over time, the repeated nature of the reciprocal game and the continuous reinforcement, train the player to adopt a new strategy, ultimately changing their behavior. This is similar to how Player A and B in the Prisoner's Dilemma develop a cooperative behavior and become friends after playing the game a sufficient… or infinitely long, number of times.

Season 3: Static Games with Asymmetric Information

Asymmetric information, also known as *incomplete information*, is a situation in which **at least one player in a game has private information that other players do not have.** This contrasts with the static games we discussed in Season 1, which are actually symmetric or complete information games, where all players have perfect knowledge of the game's rules, available strategies for every player, and exact payoffs for every possible outcome.

- The original static game

		Hunter B	
		Hunt Stag	Hunt Hare
Hunter A	Hunt Stag	**(4, 4)**	(0, 3)
	Hunt Hare	(3, 0)	**(3, 3)**

Figure B.1 Hunter game in symmetric information setting

This payoff matrix represents a complete information static game. Each player knows the other's payoff for every combination of actions, such as (4, 4). However, with asymmetric information, a player's knowledge about the game is limited. For example, **a player might know their own payoff but not the payoff of their opponent**, which can be represented as (4, □) or (□, 4).

- Static game with asymmetric information

		Hunter B	
		Hunt Stag	Hunt Hare
Hunter A	Hunt Stag	**(4, □)**	(0, □)
	Hunt Hare	(3, □)	**(3, □)**

Figure B.2 Hunter A's perspective

		Hunter B	
		Hunt Stag	Hunt Hare
Hunter A	Hunt Stag	**(□, 4)**	(□, 3)
	Hunt Hare	(□, 0)	**(□, 3)**

Figure B.3 Hunter B's perspective

This imbalance of knowledge fundamentally changes how players make decisions and the outcomes of a game. **Players must form beliefs about the unknown information** (like their opponents' payoffs and strategies, often by using probabilities) **and base their choices on those beliefs.** This type of strategic scenario is called a **Bayesian game**, and a prime example of a Bayesian game is an "auction," where players or "bidders" have incomplete information.

WHAT IS AUCTION?

An **auction** is a strategic scenario in which bidders' decisions are driven by their own **private information** and their beliefs about the private information of others.

The core of an auction is that each bidder has their personal "**true value**" for the item being sold — this is their private information. No other bidder knows this value. This **asymmetric information** is what makes auction theory a powerful application of Bayesian games.

When a bidder formulates their strategy (e.g. decides how much to bid), they cannot simply act as if they know their opponents' valuations. Instead, they must form beliefs about the distribution of other bidders' values to calculate their **expected payoff** for each possible bid, aiming to maximize their utility (payoff). This process of **using a known probability distribution to reason about an opponent's unknown valuation** is the defining feature of a **Bayesian game.**

In an auction, there is typically one seller and many buyers. This creates competition among the buyers, who must consider two key questions:

1. **Win or lose:** Will my bid be high enough to win the item?
2. **Payoff:** If I win, will my payoff be positive?

Unlike static games with complete information where players can communicate to find a "middle point" and cooperate to reach a Win-Win outcome, an auction is a one-time game with **asymmetric information**. The final outcome is not determined by a one-on-one negotiation between a buyer and seller, but rather by buyers simultaneously setting their bid prices based on their private information and their beliefs about others. This competition among buyers determines the final outcome, regardless of the seller's initial bargaining power. Different auction formats lead to different **Bayesian Nash Equilibrium**.

TYPES OF AUCTIONS

There are four main types of auctions: **descending-bid**, **ascending-bid**, **first-price sealed-bid**, and **second-price sealed-bid**.

While some of these are dynamic (such as descending and ascending auctions with real-time player interaction), they can be strategically analyzed as **static** games, which makes them easier to understand. For instance, a descending-bid auction can strategically be viewed as a first-price sealed-bid auction, and an ascending-bid auction can be viewed as a second-price sealed-bid auction. We will explore these in detail in the following episodes.

EPISODE 3.1 - Pay What You Bid 🥇

Descending-bid and First-price sealed-bid Auction

Leo had been waiting for this moment all week. A rare concert poster, signed by his favorite singer, *Brono Mues*, just showed up on Facebuck Marketplace. Within minutes, the post was flooded with comments — over a hundred people wanted it.

The seller, overwhelmed, announced:

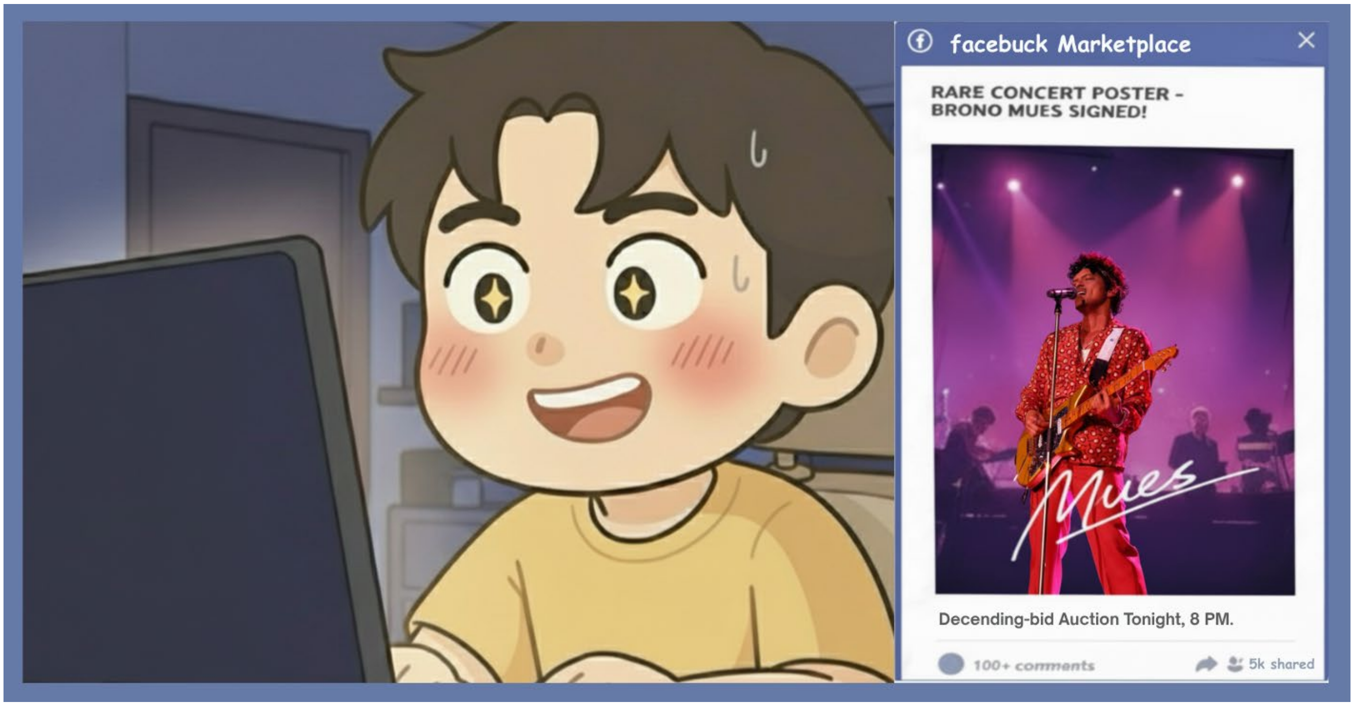
"Let's do this fair. Auction tonight, 8 PM. Highest bidder gets the poster and pays that bid price."

Leo's heart raced. He wanted this poster more than anything, but how much should he bid?

If he bid too high, he might win… but he'd also risk pushing up the second-highest price. If it went beyond his budget, he would end up borrowing from George just to pay. The poster wouldn't feel like a treasure anymore… it would come with stress and regret, a negative payoff.

But if he bid too low, he would lose. Someone else would walk away with Brono Mues's signature while Leo sat empty-handed, stuck with a payoff of zero.

It was a game of strategy. And at 8 PM sharp, *the first-price auction would begin.*



GAME MODELING

The First-Price Auction Strategy

From a game theory perspective, the goal is to maximize your payoff. Let's define our terms for Leo:

- v_i is the true value of player i $\rightarrow$ v_{Leo} is Leo's true value for the poster.
- b_i is the bid price of player i $\rightarrow$ b_{Leo} is Leo's bid price.

In a first-price auction, if a player's bid (b_i) is the winning bid (the highest), their payoff is $v_i - b_i$. If their bid is not the highest — they lose, their payoff is 0.

Let's assume Leo's true value for the poster is **$10**.

If Leo bids his true value ($b_{Leo} = v_{Leo} = 10$), he gets a payoff of 0, whether win or lose: If he wins, his payoff is 10-10 = 0. If he loses, it is also 0. So, the way to bid is **to lower the bid price slightly from your true value ($b_{Leo} < v_{Leo}$).**

By doing this, Leo will get a positive payoff if b_{Leo} is high enough to win.

This is the strategic trade-off. To win, Leo needs to bid high, but to get a large positive payoff, he needs to bid low. For example:

- Case 1: If Leo bids too close to his true value, at b_{Leo} = 9, he could win, but he will not get a very large payoff (10–9 = 1).

> However, bidders must consider two key questions:
>
> 1. **Win or lose:** Will my bid be high enough to win the item?
> 2. **Payoff:** If I win, will my payoff be positive?

- Case 2: If he bids too far below his true value, at b_{Leo} = 7, he increases his potential payoff (10–7 = 3) but would significantly reduce his chances of winning.

Since Leo does not know his opponents' true values, he must find the optimal balance between these two factors. The solution or strategy to this problem is the **Bayesian Nash Equilibrium.** In a first-price sealed-bid auction, Bayesian Equilibrium is for each player to bid **less than their true value**.

However, **the exact amount that a player shaves off their bid depends on their beliefs about the distribution of their opponents' values.**

For example, if Leo believes his opponents will value the item at a high price, he would shade his bid down only a little. On the other hand, if he thinks others value the item at a low price, he would shade the bid lower by more to get a higher payoff. This Bayesian equilibrium strategy forces a bidder to **balance the probability of winning with the payoff if they win**, ultimately maximizing their expected utility. This complex strategic decision is a **dilemma** for bidders.

Descending-bid auction

This is also called a Dutch auction. In a descending-bid auction, players interact with each other *in real-time*, which is a characteristic of a dynamic game. The seller gradually *lowers the price from a high initial value* until a bidder accepts to pay at the current price. At that moment, *the first bidder who accepts wins* the item and *pays that price.*

First-price sealed-bid auction

This is a static, simultaneous-move auction. All bidders submit their bids in a “sealed envelope” to the seller *at the same time.* After all the bids are collected, the seller opens them. The *highest bidder wins* the item and *pays the exact price that they bid.*

Why a Descending-bid Auction can be viewed as a First-price Auction

A descending-bid auction is a dynamic game, where players interact in real-time, while a first-price sealed-bid auction is a static game where players move simultaneously. However, from a game theory perspective, they are strategically equivalent.

The reason for this is that in a descending-bid auction, a rational bidder's strategy can be reduced to a single decision: determining their **stopping price.**

If they stop the bid price where it is equal to their true value ($b_i = v_i$), they get a payoff of 0 whether they win or lose. So, their **stopping price** should be slightly lower than their true value. After bidders decide on the maximum price they are willing to pay, they should commit to accepting the price **as soon as the seller lowers it to that point.**

- If they accept before the price is lowered to this predetermined value, they could win, but they will not get a very large payoff
- If they wait for it to drop any further, they risk another bidder stopping the auction first and losing the item.

This single, pre-determined decision involves the exact **same strategic trade-off** that Leo faced **in the first-price auction**: a bidder must **balance** the need to bid high enough to win with the goal of bidding low enough to get a large positive payoff. This **complex decision** in a descending-bid and a first-price auction **creates a dilemma for players, making them be the inferior option** for the seller and bidders, **compared to a second-price auction.**

The Bayesian Nash Equilibrium for both a descending-bid and a first-price sealed-bid auction is for each player to bid (or set a stopping price) that is less than their true value.

The Cost of Winning

In any auction, the primary goal is to win. However, winning can sometimes be a strategic trap. Whether a player pays their own highest bid (First-Price Bid Auction) or the second-highest bid (Second-Price Auction), having many competitors often leads to the **Winner's Curse**.

Because players often have different information or varying levels of optimism, they are likely to overestimate the true market value of an item. To ensure victory, a player may overbid, ending up with a price higher than the item's actual value. If that player intends to **resale** the item later, they will likely incur a loss relative to the price they paid.

GAME APPLICATION

Application

Real-World Example: Government Infrastructure Tenders

This is the most standard use of the first-price sealed-bid auction. When a government wants to build a new bridge or highway, they invite private companies to submit *bids*, in this case, it is the price they will charge to do the work. Companies must bid low enough to win the contract, but high enough to cover their costs and make a profit. However, because the bids are hidden, the winner often experiences the **Winner's Curse**.

Imagine company *A* bids \$10 million and the next closest competitor bids \$12 million. Since the government always picks the lowest cost, *A* wins. However, the winner realizes he could have bid \$11.9 million and still won the contract. By bidding \$10 million, he "wasted" \$2 million in potential profit.

To avoid it, bidders try to "shade" their bids by bidding lower than their own true value. However, in a crowded auction, the person who shades the least, or overestimates the value the most — often wins and overpays.

EPISODE 3.2 - Pay What You Don't Bid 🥈

Ascending-bid and Second-price sealed-bid Auction

Three hours before the much-hyped auction, the seller suddenly changed the rules: *"Update! The auction will now be a second-price sealed-bid auction. Highest bidder wins, but only pays the second-highest price."*

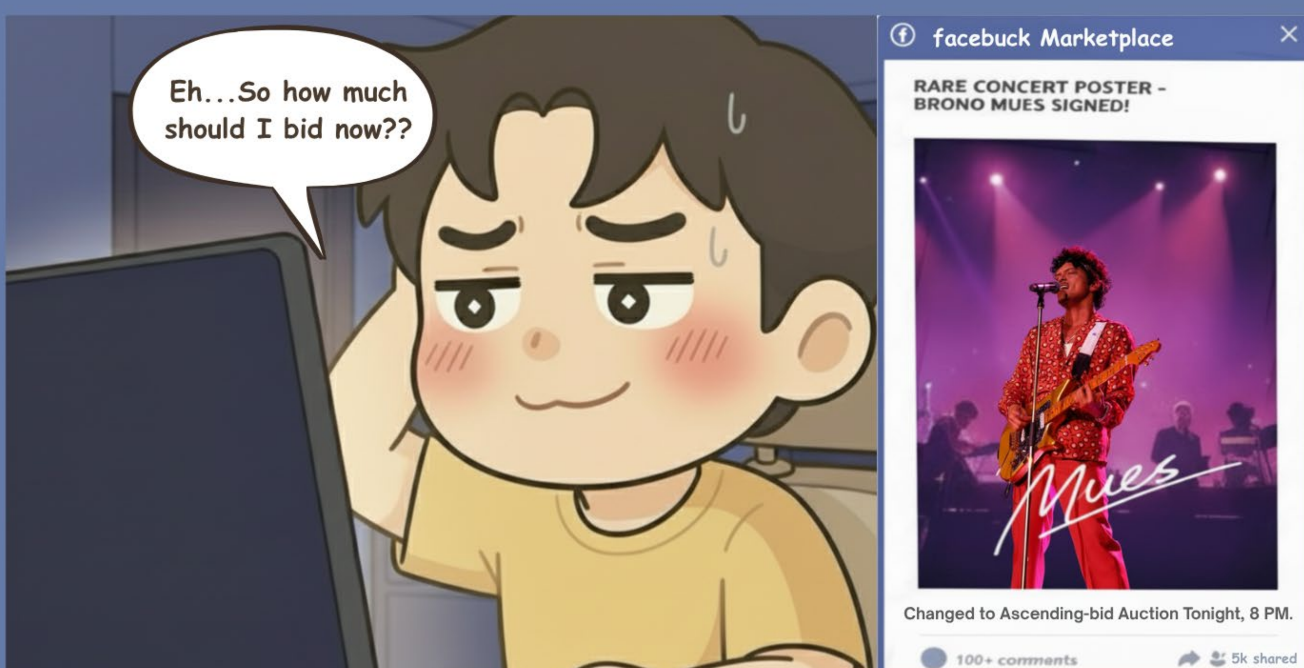
Some people grumbled in the comments, but no one really protested. After all, the goal was the same — try to win by using the strategy that maximizes their payoff.

For Leo, however, this change made his head spin.

"What's the right strategy now?" he muttered.

Again, Leo wants to win, but he doesn't know how much he should bid. Different auctions have different strategies, leading to different **Bayesian Nash Equilibrium**.

Another game of strategy. The countdown to the auction was on.



GAME MODELING

Second-price Auction Strategy

Let v_{Leo} is Leo's true value, and b_{Leo} is Leo's bid price.

- If b_{Leo} is the winning bid (the highest), and b_{2nd} is the second-highest bid from another player which is the price Leo needs to pay, then Leo's payoff is $v_{Leo} - b_{2nd}$.
- If Leo's bid is not the highest, he loses, his payoff is 0.

Assume Leo's true value for the poster is **$10**.

If Leo bids his true value ($b_{Leo} = v_{Leo} = 10$), he pays at b_{2nd} , which is unknown value based on another player. The key is **b_{Leo} only affects whether Leo wins or loses, but it never affects how much Leo pays in the event that he wins,** as the second-highest bid comes from another player. Thus, **the way to bid is to simply bid your true value ($b_{Leo} = v_{Leo}$).**

Since Leo's true value is $10, his best strategy is to bid $10.

1. If Leo bids anything less than 10 (e.g. bids 8), $b_{Leo} < v_{Leo}$:
   - Case 1.1: $b_{Leo} = 8$ is still the highest. Leo wins, and his payoff is still 10 - b_{2nd}. He did not lose anything by bidding lower in this case.

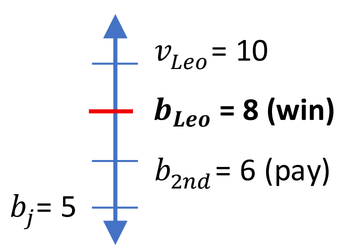


Figure 3.2.1 Case 1.1

- Case 1.2: If $b_{Leo} = 8$ is *not* the highest and b_{2nd} > 10.

  Leo loses and gets a payoff of 0. However, it is better than a negative payoff. This is the same as the case he had bid $b_{Leo} = v_{Leo}$ but lost.

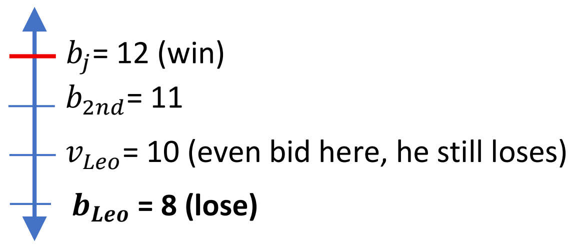


Figure 3.2.2 Case 1.2

- Case 1.3: There is *the worst case* that the second-highest bid is between b_{Leo} and v_{Leo} (e.g. $b_{2nd} = 9$). Leo would have won if he bided his true value, $b_{Leo} = v_{Leo} = 10$, also gotten a positive payoff of 10-9 = 1. But as he bids b_{Leo} = 8, Leo loses and gets 0 instead.

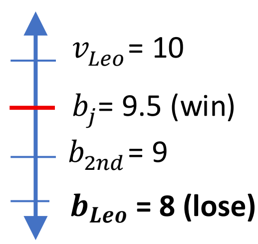


Figure 3.2.3 Case 1.3

In conclusion, bidding $\boldsymbol{b_i < v_i}$ **means a player risks losing the item to a bid of player *j* that is between his lower bid** (8) **and his true value** (10), and he would miss out on a positive payoff.

2. If Leo bids more than 10 (e.g. bids 15), $b_{Leo} > v_{Leo}$:

- Case 2.1: $b_{Leo} = 15$ is the highest and $b_{2nd} < 10$. He pays the second-highest bid which is lower than his true value. His payoff is $10 - b_{2nd}$, a lucky positive payoff.

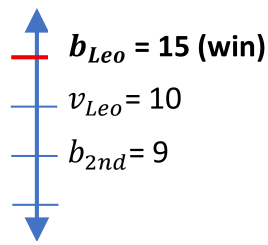


Figure 3.2.4 Case 2.1

- Case 2.2: The worst case is that the second-highest bid is over v_{Leo} (e.g. b_{2nd} = 11). Leo wins, but his payoff is negative (-1). If he had bid his true value of 10, he would just have lost and got 0, which is better.

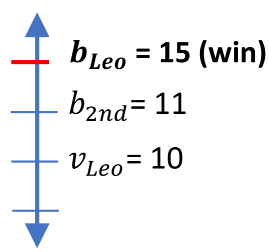


Figure 3.2.5 Case 2.2

So, bidding $\boldsymbol{b_i} > \boldsymbol{v_i}$ can lead to a negative payoff, if the second-highest bid is also over $\boldsymbol{v_i}$.

In conclusion, bidding true value never results in a worse payoff than bidding higher or lower. It is the best move, regardless of what other bidders do. Thus, **truthful bidding** is players' **dominant strategy** and the Bayesian Nash Equilibrium in a second-price auction.

Since the price the winner pays is determined by the second-highest bid which is not his own bid, the winner does not need to worry about shading the bid down to get a positive payoff. **The payoff comes automatically if player *truly values* the item the most.**

Ascending-bid auction

This is also known as an *English auction*. Ascending-bid auction is a dynamic game where players interact *in real-time*. The seller gradually *raises* the price from *a low initial value to high*. Bidders signal their interest by raising their hands or calling out bids, and they drop out as the price increases. The auction ends when *only one bidder is left*, who is the winner and must *pay at the second-to-last bidder dropped out.*

Second-price sealed-bid auction

This is a static, *simultaneous-move* auction. All bidders submit their bids in a sealed envelope at the same time. After all the bids are collected, the seller opens them. The *highest bidder wins* but *pays* the price of the *second-highest bid*.

Why an Ascending-bid Auction can be viewed as a Second-price Auction

An ascending-bid auction is strategically equivalent to a **second-price sealed-bid auction**. The reason is that a rational bidder's optimal strategy in an ascending-bid auction can be reduced to a single decision: determining their **stopping price.** A bidder should stay in the auction and continue to raise their hand as long as the price is below their true value, and stop bidding the moment the price exceeds their true value. This strategy is optimal for two reasons:

- If a bidder stops bidding before the price reaches their true value, they risk losing the item to a bid that is still below what they are willing to pay, thus missing out on a potential positive payoff. (Case 1.3)
- If they continue bidding past their true value and win, they have to pay more than they believe the item is worth, resulting in a negative payoff. (Case 2.2)

Therefore, the most logical strategy is to bid up to your true value, then stop. This decision is strategically identical to the **dominant strategy** in a second-price sealed-bid auction, which is to simply bid your true value. In both cases, the winner pays a price determined by the second-highest bid.

Seller Revenue

A common question is whether a seller will earn more from a first-price or a second-price auction. A surprising result from the **Revenue Equivalence Theorem**, shows that under certain assumptions, the seller's expected revenue is exactly the same in both auction types. This may seem counterintuitive. The first-price auction seems like it would generate a higher price, while the second-price auction feels like a bargain for the winner. However, the difference in bidder strategy perfectly balances this out.

GAME TRANSFORMATION

Which Auction is Best?

The core difference between the two auction types lies in the bidder's strategy:

- **First-price auction:** Bidding is a complex strategic decision. Bidders must make an educated guess about their opponents' valuations and calculate how much to **shade their bid down from their true value.** If a player shades too much, they risk losing the item. If they do not shade enough, they lose out on a potential payoff. This makes the first-price auction a more aggressive and high-stakes game.

- **Second-price auction:** The best move for every player is to simply bid their **true value**. Since the winner pays the second-highest bid, the price they pay is independent of their own bid. This simplifies the decision-making process, as bidders do not need to worry about the complex trade-off of balancing their bid price with the probability of winning.

Although the seller's expected revenue is theoretically the same, the **second-price auction** is often considered a superior design from a practical standpoint. It is a **more efficient and straightforward** process for both the seller and the bidders. Bidders do not need to engage in complex strategic thinking and can simply bid their true value, making the auction easier and less stressful, **no more dilemma**. This simplicity also ensures fairness, as the person who truly values the item the most will always win.

Application

Real-World Example: eBay and Google Ads

While Second-Price auctions are less common in manual government tenders, they dominate the digital world:

- **eBay's Proxy Bidding:** When you enter the maximum amount you are willing to pay for a collectible, eBay's system only bids enough to outbid the previous person. If your *true value* is $100 but the next highest bidder only bid $50, you win the item for approximately $51. This is effectively a second-price auction.
- **Google Search Ads:** When you search for *best running shoes*, multiple companies bid to show you an ad. Google uses a modified second-price auction where the winner pays just enough to beat the "Ad Rank" of the bidder below them.

Ultimately, in any auction format, the winner never knows exactly how much they **overpaid** until the other bids are revealed, making the strategy a delicate balance between aggression and caution.

EPISODE 3.3 - Pay Even If You Lose 💰

All-pay auction (Advanced)

The four main types of auctions — descending, first-price, ascending, and second-price — and their respective strategies have been discussed.

However, there is another advanced type of auction, commonly found in *political lobbying*, called the **all-pay auction**.

GAME MODELING

All-pay auction

The all-pay auction is a more **complex version of the first-price auction.** The *highest bidder wins* and gets the item, paying the price that he or she bid. The crucial difference, however, is that **all bidders who participate in the auction must pay their bid, regardless of whether they win or not**.

The Most Uncertain Auction

An all-pay auction creates more uncertainty for bidders than any other auction format. Unlike a first-price and a second-price auction, where a player can use a clear strategy: bidding slightly below their true value and truthful bidding, the all-pay auction is far more *complex and unpredictable,* because every bidder must pay their bid, regardless of whether they win or lose. This unique rule eliminates any simple best move. For a player, the main strategic dilemma is **not just about winning or losing, but about the risk of paying and getting nothing in return**.

The Logic of a Mixed Strategy

In a game with this much uncertainty, a player cannot have a single best strategy. Since bidders now do not know the private values of their opponents, if a bidder always chooses the same bid price (pure strategy), their opponents could eventually figure out that bidder's strategy or true value and exploit it. The best way to prevent others from guessing their private value is to randomize their bid, making the private value unpredictable. This mirrors the concept of **mixed strategy equilibrium** in static games with complete information, where a pure strategy also may not exist. Bidders will not choose one specific bid price, but rather they choose from a range of possible bid prices according to a probability distribution.

Therefore, **Bayesian Nash Equilibrium of all-pay auction** is **a mixed strategy.** Every rational player will **randomly choose their bid**, resulting in the high level of uncertainty and chaos.

The Hunter game, with asymmetric information, can be viewed as an all-pay auction since both has no pure strategy and only a mixed equilibrium. Actually, the equilibrium is the same as the mixed equilibrium of a complete information game.

		Hunter B	
		Hunt Stag (q)	Hunt Hare (1-q)
Hunter A	Hunt Stag	**(4, □)**	(0, □)
	Hunt Hare	(3, □)	**(3, □)**

Figure 3.3.1 Hunter A's perspective: Whether B choose to *bid* Stag or Hare, A will get the same expected payoff.

		Hunter B	
		Hunt Stag	Hunt Hare
Hunter A	Hunt Stag (p)	**(□, 4)**	(□, 3)
	Hunt Hare (1-p)	(□, 0)	**(□, 3)**

Figure 3.3.2 Hunter B's perspective: Whether A choose to *bid* Stag or Hare, B will get the same expected payoff.

To find this equilibrium in the Hunter game, we solve for the probabilities that make each player indifferent between their choices.

- For Bidder A to be indifferent between B's bidding Stag or Hare, the expected payoff must be equal: $4q + 0(1-q) = 3q + 3(1-q)$
- For Bidder B to be indifferent between A's bidding Stag or Hare, the expected payoff must be equal: $4p + 0(1-p) = 3q + 3(1-q)$

Solving this gives us p, q = 3/4

The Bayesian Nash equilibrium using mixed strategy is for each bidder to *bid* Stag with a probability of 3/4, bid Hare with a probability of 1/4. We also can use a mixed strategy as a **Bayesian Equilibrium** for n bidders and n choices of bid prices in an all-pay auction. This strategy allows a bidder to balance the cost of bidding with the chance of winning, while making sure their own private information cannot be anticipated.

Seller Revenue and All-pay Auction

The **Revenue Equivalence Theorem** states that a seller's expected revenue is theoretically the same regardless of whether they hold a first-price, second-price, or all-pay auction. A seller can choose a more complex format like an all-pay auction, but they will not earn more.

GAME TRANSFORMATION

Which Auction is Best?

Although the seller's expected revenue is exactly the same in all three auction types, it does not mean all of them are equally good for the seller.

As mentioned in the previous episode, **the second-price auction is often considered the best choice.** While the revenue remains the same, the second-price auction is more efficient and easier for bidders and seller. **It simplifies the strategic dilemma for buyers, who can bid their true value without worrying about complex calculations**. This transparency makes the auction more appealing to a wider range of bidders and ensures that the item is allocated to the person who truly values it the most, leading to a more efficient market outcome.

Application

Real-World Example: Political lobbying

This is a perfect example of an all-pay auction. Every side has to *spend* money, time, or other resources to influence a targeted politician, but only the successful side receives the benefit from their investment. All other groups that participated have to pay their costs without receiving any benefit. This makes the all-pay auction an extremely high-stakes game.

EPISODE 3.4 - Don't Tell Sara 🎟️

Matching Market (Multiple-Item Auction)

Ever since that night when George, Leo, and Sara ordered a late-night meal from a delivery app together, something has changed. George and Sara started texting. And before long… they were dating!

Now, George is plotting the perfect surprise. He wants to officially ask Sara to be his girlfriend. His plan? Two concert tickets, so they can scream, sing, and laugh together.

But there's a catch: The three hottest concerts, **Green Pink**, **Brono Mues**, and **Tanya Switt**, sold out ages ago. The only way left is to buy tickets second-hand from resellers… and since these sellers can freely set their own prices, they are running their own auctions. Each seller lists their pair of tickets online with a low starting price, keeping the sale open for few days.

If there are multiple buyers, the seller raises the price higher and higher, like an **ascending bid**. Interested buyers stay in. While the price climbs up… until only one buyer remains — the winner — who pays the final highest price.

George is nervous, but determined. He values Green Pink the most as it is Sara's favorite girl group, but Brono Mues or Tanya Switt would still be good options.

Meanwhile, **Sara… not knowing George is secretly planning this surprise**, also joins the auction for Green Pink tickets herself.

And then there's Leo. He happens to catch George scrolling nervously through the auction site. Wanting to help his best friend's romance succeed, Leo decides to join the auction too. Not for himself, but for George and Sara.

Since Brono Mues is his idol, Leo especially wants to get *those* tickets for George and Sara.

So, the stage is set: three buyers, three pair of different concert tickets… this is multiple item auctions.

Who wins what? And will George's big surprise plan work out?

GAME MODELING

Concert Tickets in Matching Market

Buyer Valuations:

- George values *a pair of tickets* at: Green Pink = 8 (Sara's favorite), Brono Mues = 6, Tanya Switt = 4
- Sara values: Green Pink = 9 ("I must have this!") Brono Mues = 5, Tanya Switt = 6
- Leo values: Green Pink = 7, Brono Mues = 8 (best singer in his view), Tanya Switt = 5

This game is a static game with asymmetric information. **George, Sara, and Leo do not know each other after entering the auction game**; each player only knows their own private valuation, while the private values and strategies of their opponents are unknown. However, since this multiple-item auction is similar to an ascending-bid auction, **sellers will first announce their initial valuation**, or "cost" (if the seller sells at a price lower than this, they incur a loss).

In the matching market, we have to find out the **market-clearing price**, which means the price that allows each person to get the concert tickets that give them the highest payoff, while making sure everyone gets tickets.

<u>Step 1:</u> Rank player's valuations for each concert tickets.

- George values Green Pink the most: 8 points.
- Sara's favorite is Green Pink too: 9 points.
- Leo prefers Brono Mues: 8 points.

Step 2: Find out the market-clearing price, by calculating each player payoffs for each ticket.

- George's highest payoff: Green Pink (4).
- Sara's highest payoff: Green Pink (5).
- Leo's highest payoff: Brono Mues (5).

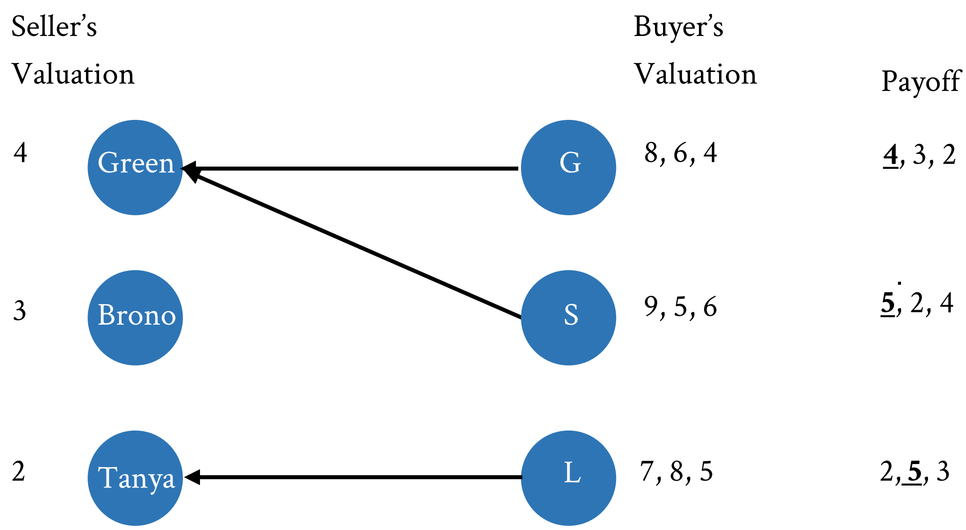


Figure 3.4.1 Market in Round 1

In round 1, both George and Sara rank Green Pink as their top choice — Green Pink are in high demand, but only one unit (*a pair of tickets*) is available. So, it is not allowed for both to choose it. The price of 4 (Green), 3 (Brono), 2 (Tanya)is not a market-clearing price. That means we need to adjust by increasing the price of Green Pink by 1.

In round 2, we raise the price of Green Pink from 4 to 5.

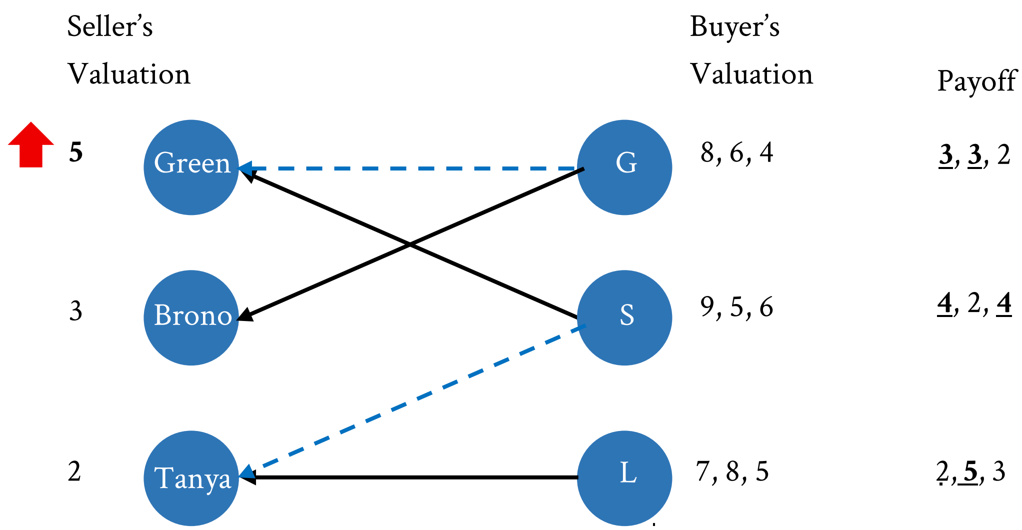


Figure 3.4.2 Market in Round 2

- George's highest payoff: Green Pink (3) and Brono Mues (3).
- Sara's highest payoff: Green Pink (4) and Tanya Switt (4).
- Leo's highest payoff: Brono Mues (5).

Nash equilibrium (Matching Market)

We find the market-clearing price is 5 (Green), 3 (Brono), 2 (Tanya). Now, we can assign each concert tickets to one person, and everyone gets the tickets that gives them their highest payoff.

- George gets Tanya Switt with a payoff of 3.
- Sara gets Green Pink with a payoff of 4.
- Leo gets Brono Mues with a payoff of 5.

Everyone gets tickets — and the market is cleared!

Few days later, George handed over Tanya Switt tickets to Sara, his expression a little sheepish.

“Honestly, I wanted to take you to the Green Pink concert… but I only managed to win Tanya Switt. So… let’s go together anyway. I’ll buy you flowers to make up for it.”

Sara burst out laughing and jumped into his arms.

“Silly! I already won Green Pink tickets myself! That means… we can go to both concerts!”

The two of them spun around in excitement, their laughter echoing down the dormitory.

Meanwhile, on the other side of the room, Leo sighed and texted Tom: “Hey… wanna go to Brono Mues with me, bro?”

At Green Pink concert, George and Sara cheered and danced. While Leo endured Tom’s booming voice singing every Brono Mues lyric directly into his ear.

Maybe it’s time I find a girlfriend too, Leo thought.

Matching Market

A **matching market** is a system for pairing buyers with items or with sellers based on a buyer’s preferences, as reflected in their valuation for each item. This game is designed to create a stable and efficient outcome when multiple items are available and buyers have conflicting preferences, such as competing for the same item.

In a matching market, the final price of an item is not determined by a single bid but through an economic mechanism that finds the **market-clearing price**. This is the price at which every item is claimed, the supply and demand for each item are perfectly matched, and every buyer gets the item they value most.

The market-clearing price can be found by adjusting the price of the highly sought-after item until the contention was resolved. In the concert tickets example, the price of Green Pink was raised until George and Sara's choices were no longer in conflict and the graph show perfect match between sellers and buyers. The process of finding this market-clearing **equilibrium** ensures that:

1. **Every item is matched** to a buyer.
2. **Every buyer receives an item** that gives them their highest possible payoff.

The final outcome is a stable equilibrium where no individual has an incentive to change their choice.

Multiple-Item Auction

A multiple-item auction is a type of auction where **several items are sold at the same time.** This differs from a single-item auction, like those discussed in previous chapters, where bidders compete for only one item. In a multiple-item auction, the goal is to efficiently **allocate all the items to the bidders who value them the most.**

The most important thing to remember: this market game is still played under the condition that no one knows each other's private value, not even knowing who their opponents are.

The process for this type of auction is similar to an ascending-bid auction. The seller gradually raises the price, and bidders drop out as the price increases. Under a matching market setting, this multiple-item auction ends when **all of the items have been claimed by different buyers**. Unlike a standard ascending-bid auction, the winners here must pay the highest value they bid, not the price at which the second-to-last bidder dropped out. Overall, a multiple-item auction is **like playing an ascending-bid auction for each item one by one until all of the items have been claimed by different people.**

The concert tickets scenario is a great example of a multiple-item auction where a **matching market mechanism** is used to determine the final allocation and price. Instead of simply having a bidding war for each concert tickets, the process relies on the bidders' valuations to find a price that clears the market for all three items **simultaneously.** This ensures that the final result is a fair and stable match, even when there are many choices and conflicting preferences.

GAME TRANSFORMATION

Winning through Value

This matching market itself begins with a classic dilemma: a resource conflict where two buyers (George and Sara) aim for the same single rare item (a pair of Green Pink tickets). However, at the market equilibrium point, this conflict is resolved by increasing the item's price. This forces players to reveal their true valuations. If a player truly values an item the most, they will be willing to **sacrifice more resources, such as money, effort and time, to obtain it** even at a higher price.

Once the price is increased until only one buyer is left as the winner, the market conflict is resolved. This actually reflects the realistic market mechanism, in which the item is automatically delivered to the participant with the highest valuation.

Application

Real-World Example: Dating App Ecosystems

Dating apps like *Tinder* serve as a practical example of matching markets where the *transaction* is based on **mutual preference** rather than a monetary price. However, because this is a *two-sided market*, unlike a ticket bidding scenario where the highest price wins, a connection can only be established if both parties "swipe right," meaning the preference must be mutual and both sides must accept the match.

In this ecosystem, a "rare item" is a user with many potential matches — a profile that is highly sought after by the network. To secure a match with such a desirable partner, a buyer (another user) must commit more resources. In the absence of a monetary price, these resources take the form of **effort and time**. This might involve spending more time crafting a better profile, putting more effort into a personalized first message, or even paying for premium features (like a Super Like) to stand out from the crowd.

Just like the concert ticket auction, the conflict for a popular profile is resolved through this commitment. The “match” is eventually established with the user who is willing to provide the highest level of effort and preference. This highlights how more demand is always better for the central nodes, as it increases their bargaining power and ensures that the final match is made with the person who values the connection the most.

Season 4: Dynamic Games with Asymmetric Information

Markets with asymmetric information often reflect individuals' beliefs and expectations about their opponents' private information. However, since players have incomplete knowledge about the game, in dynamic games, while players make decisions sequentially, they actually do not know the final outcome. The equilibrium is therefore influenced by what each player expects to happen. This belief can lead to a positive outcome, or, like in the traffic game, it can lead to a special case of **Braess's Paradox**. Crucially, the unknown value of the outcome depends not just on one player, but on what every player in the market believes.

INFORMATION IMBALANCE ISSUES – Adverse Selection vs. Moral Hazard

In many dynamic settings where buyers and sellers interact, one side of the market normally has better information about products than the other side does. This information imbalance creates two issues that can potentially lead to **market failure**.

1. **Adverse Selection :** A phenomenon caused by **hidden characteristics** of the item that the better-informed party has and are detrimental to the less-informed party. This problem occurs **before the transaction.**
   - **Seller Advantage – Hidden Quality:** In the market for **used cars**, the seller knows the true quality of the car (whether it's a good car or a "**lemon**"), but the buyer does not.
   - **Buyer Advantage – Hidden Risk:** When selling **health insurance**, buyers know their personal health risk in detail more than the sellers do. This risk determines the true likelihood of a claim, which essentially reflects the true cost (or 'value') of the insurance from the insurer's perspective.

2. **Moral Hazard:** Occurs **after the transaction**, when one party puts in less effort or takes more risks because they know the other party will bear the cost of those actions for them.
   - **Post-Insurance Behavior:** A driver with full car insurance may drive less carefully, knowing the insurer will cover any potential damages.

THE TRUST DILEMMA: Symmetric and Asymmetric Information

In Seasons 3 and 4, dilemmas arise primarily from Asymmetric Information, where players possess incomplete or unequal knowledge about the market. This creates a higher-level dilemma rooted fundamentally in **Trust**.

Evolution of the Dilemma: Seasons 1-2 vs. Seasons 3-4

We can categorize the challenges of Game Theory into two types

1. **Original Dilemmas (Seasons 1-2):** These occurred in Complete Information games where all players knew the payoffs, but the structure of the game led to poor outcomes, such as uncoordinated results in Coordination game, inferior outcomes in the Hunter game, or Braess's Paradox in network traffic.
    - The Limit of Trust: While some structural dilemmas (like Braess's Paradox or Intermediary Market issues) cannot be solved by trust alone, most symmetric games (such as the Prisoner's Dilemma) are still resolved if players trust one another.

2. **Trust Dilemmas (Seasons 3-4):** These occur because players have incomplete information. When one player knows more than the other, the game is no longer just about strategy; it is about the **how they trust in the information other players provided**. Just like the strategic games of earlier seasons, these informational games are most effectively resolved when players establish a foundation of trust.

Ultimately, in almost every game, whether **symmetric or asymmetric**, the core conflict boils down to a **Trust Issue**. We consider trust to be a "higher-level" dilemma. Solving it is our **first principle for establishing cooperation**. If players can build a mechanism for trust, they can reach the Win-Win Nash Equilibrium.

The Trust Fee by a Third-Party

This trust issue happens in many everyday perspectives. Consider the **airport taxi example** again, a driver may cheat a tourist because they see the interaction as a one-time game. Since you are not a local, there is no future trip where you would use their service again, tempting them to increase the price without your realization.

However, today, you have a choice: you can use Uber. While Uber's price may be higher than a direct taxi, that premium ensures you will not be cheated or dropped off at the wrong location. Uber's feedback and rating system effectively includes an "insurance fee" for quality. This higher price is, in reality, a **Trust Fee**.

Another example, discussed in detail in the episode 4.1, is the **Lemon Market** for used cars. Buyers often do not trust what sellers tell them about a car's type. Consequently, buyers will only pay an average price (somewhere between the value of a "good car" and a "lemon"). To avoid this, buyers may use a third-party platform instead of buying directly from sellers. These platforms charge a fee (perhaps 5–10%), but they ensure the car quality matches the price paid.

In conclusion, in any game involving a trust dilemma, players face a fundamental trade-off between risk and gain. They can either **pay Trust Fee to ensure the outcome**, or **accepting the risk of information imbalance**.

THE ADVANTAGE OF BETTER INFORMATION

The better-informed side will gain a significant advantage over the other, allowing them to capture most of the potential **profit surplus**. For example, if buyers know the seller's value (cost), they can set their bid just slightly higher than the seller's cost. Conversely, if sellers know the buyer's valuation, they can set their asking price just below the buyer's value.

However, in a general asymmetric setting where both sides are unsure of their opponent's private value, the market price is a reflection of what the **less-informed side** collectively believes or expects about the average quality of the product.

EPISODE 4.1 - Shiny on the Outside 🚘

Adverse Selection – The Market for Lemons

After four years of study, with sweat and endless part-time jobs, George finally saved enough to buy himself a graduation gift: a car of his own!

Excited, he dragged Leo along to a nearby used-car showroom. Rows of shiny cars waited for them, but neither George nor Leo knew a thing about cars.

"Bro… how do we even know which one's good?" Leo whispered.

George sighed. "No clue. They all look… car-ish."

Here's the catch: there are two kinds of used cars. Good cars: worth the money. Or bad cars, so-called *lemons*: shiny on the outside, but full of hidden problems. The point is, George and Leo can't tell which is which. **The seller knows more than the buyers, while buyers are stuck with uncertainty**.

Will George get a sweet ride that's worth every dollar? Or will he drive home in a lemon… the shiny trap of the used-car world?

GAME MODELING

The Lemon Dilemma

Sellers' Values: Good cars \$10k, Lemon \$4k. These are seller's private valuations. Sellers are willing to sell their car for any price equal to or greater than their value.

Buyers' Values: Good cars \$12k, Lemon \$5k. These are buyer's private valuations. They are willing to pay their expected value of a car.

<u>Assume that:</u>

- The sellers have better information about products than buyer does. They know the quality of their cars
- Buyers, including George, cannot tell the difference between a good car and a lemon. Because of this, **there can only be one price for all used cars on the market.** This single price reflects the **buyers' expected value**, which is calculated based on the fraction of good cars they believe are in the market.
- There is more demand than supply of use cars.

Let g be the fraction of good cars in the market.

Sellers know exactly what fraction g is, while Buyers have only "expectation" of g.

<u>Find the value that buyers place for all used car</u>

Buyers' expected value = (Fraction of good x buyer's value for good cars) + (Fraction of lemon x buyer's value for lemons)

$= 12g + 5(1-g)$

$= 7g + 5$

This leads to two possible Nash equilibrium

1. An equilibrium where **all types of cars are offered for sale.**

To make sellers offer their cars for sale, the market price must be higher than seller's value (cost), which means at least \$10k, so sellers are willing to sell both his good cars and lemon.

$$7g + 5 \geq 10$$

$$g \geq 5/7$$

- If $g \geq 5/7$, means the fraction of good cars is *high enough* to make buyer have confidence and are willing to pay \$10k or more:
  - Good cars: seller values at 10 → offer for sale.
  - Lemons: seller values at 5 → offer for sale.

  Since demand is more than supply, both good cars and lemons in the market are sold if $g \geq 5/7$. **All sellers are happy while some buyers get a lemon for an expensive price, the market functions.**

2. An equilibrium where "**market fails.**"

- If $g < 5/7$, means the fraction of good cars is *too low to instill buyer confidence.* They are only willing to pay a price below \$10k:
  - Good cars: seller values at 10 → NOT offer for sale
  - Lemons: seller values at 5 → offer for sale

  The only cars available are lemons because good cars' seller will not enter the market. The buyers' expected value drops to \$5k, which is buyers' true value. This represents the complete **market failure** because the market for good cars disappears entirely, which is *another type of dilemma.*

Adverse Selection

In a world of **symmetric information**, where the quality of all used cars (good or bad) is known to everyone, the market would be highly **efficient**. Good cars and lemons will have separate prices. Buyers and sellers could negotiate a **Nash bargaining solution** to find a mutually satisfying price for each quality level.

However, the problem arises under asymmetric information — when the buyer cannot tell **before the transaction** whether the car is good or bad. This **pre-contractual** information asymmetry is called **Adverse Selection**, it forces the less-informed party to base their price on the average quality, potentially leading to the exit of high-quality goods, then… *market failure.*

The Dilemma of Information Imbalance – Market Failure

Since the quality of the item is hidden from the less-informed party (the buyer), they cannot find a Nash bargaining solution. This leads to a situation where there is only one price for all used cars, reflecting the buyer's **expected value** and uncertainty they face.

During interaction between buyer and seller in dynamic game, if buyers receive news that increases their uncertainty about the average quality of items, they become **conservative** and lower their expected value, resulting in a dilemma: the price drops, which incentivizes sellers of high-quality goods (good cars) to exit the market. The market for good cars disappears, and the final outcome is **market failure**.

At the beginning of this book, the Prisoner's dilemma is merely an inferior outcome compared to the optimal outcome, a Win-Win one. However, in this lemon market, the dilemma is the destruction of the entire market, leaving only low-quality goods at low prices, which is a severe penalty that affects everyone.

Market Failure Ingredients

Market failure in the lemon scenario is caused by three key features:

1. **Varying Quality:** The items offered have different qualities (e.g. good cars and lemons).
2. **Buyer Valuation:** Buyers value the items at least as much as the sellers do. (e.g. a buyer's value for a good car is $12k, while a seller's value is $10k). In a market with perfect information, this would lead to a successful outcome, an efficient market.
3. **Asymmetric Information:** The most critical ingredient. Only one side has better information about the items. Because of this, all items must be sold for the same uniform market price, and sellers will only offer their items if they value them at or below this uniform price, so they can get a positive payoff.

GAME TRANSFORMATION

How to Solve the Market Failure?

To solve market inefficiency, it requires action from both sides to share or extract information:

1. Signaling – Action by the Better-Informed Party: The better-informed party (the seller in lemon market) actively **signal the quality of their product** to the less-informed party (the buyer). A signal is a costly investment to prove that the seller's car is good, such as providing a lengthy warranty, obtaining third-party certification, or investing in extensive marketing. The goal of signaling is to convince the buyer that the fraction of good products in the market is high enough to justify raising the buyer's **expected value** and, consequently, their willing-to-pay price.

The Investment Decision: A seller must calculate whether the cost of signaling is worth the potential profit, as the investment raises the car's overall cost:

- Worthwhile Investment: If the investment in signaling raises the buyer's expected value (and thus the market price) enough to cover the signaling cost and still yield maximum profit, the seller will signal.
- Unprofitable Investment: If the investment cost is high, but the buyer's expected value does not rise enough, the seller will not signal, as it will not be profitable.

Sellers must find the optimal investment level that maximizes their overall profit, even if it is difficult to determine.

2. **Screening – Action by the Less-Informed Party:** The less-informed party (the buyer) takes steps to **extract or search for more information** independently. Buyers can actively set **demanded criteria** that only high-quality sellers can realistically fulfill.

For instance, a buyer might demand an extended service contract or a guarantee that covers all major components. If the seller agrees to these demanding requirements, the buyer is willing to purchase at a higher price, as the requirements act as an assurance of quality. If the seller refuses, the buyer lowers their willing-to-pay price, assuming the product is low quality.

By employing these two methods, the market attempts to overcome a failure arising from information asymmetry, bringing the market closer **to an efficient outcome.**

Application

Real-World Example: The Job Market

The applicant is the "seller" of talent, and the company is the "buyer." The company does not know if the potential employee is a high-type or low-type talent. If the company believes there is not enough high talent in the market, they will lower the hiring price (salary).

- **Signaling – Applicant Action:** Applicants (the better-informed party) must **signal their quality** by investing in costly, hard-to-fake attributes, such as a master's degree (a 2-year time cost), diplomas, certificates, or volunteering. The worth of this investment depends on how much the company values that signal shown on their resume.
- **Screening – Employer Action:** Employers (the less-informed party) **use interviews and tests to screen applicants.** Instead of simply relying on general experience, the employer asks specific, highly technical questions (e.g. "Describe a time you solved a conflict using advanced algorithms"). Only truly high-skilled applicants can provide satisfactory answers, forcing the applicant to reveal their true competence.

Real-World Example: Geopolitical Security of China vs. US

The US (or its allies — the less-informed party) does not know the true operational readiness and combat effectiveness of China's military. If the US mistakenly assesses China to be weak and at low readiness, it will miscalculate the risk of confrontation. This may incentivize the US to become more aggressive in disputed territories, *leading to world war from incomplete information.*

- **Signaling – China's Action:** China (the better-informed party) must **signal its power** to deter the US. This is done through costly military activities. For example, the government performs large-scale exercises like missile launch drills, showcases new aircraft carriers, or announces breakthrough hypersonic missile tests. China must measure whether the massive cost of these displays is outweighed by the benefit of enhanced geopolitical standing and deterrence.
- **Screening – US Action:** The US (the less-informed party) uses Intelligence and Counter-Intelligence to **screen China's signal**. It invests in technologies (reconnaissance flights, satellite imagery, cyber penetration) to observe the drills. This screening is designed to verify the fidelity of the public claim—to determine if the missile was genuinely successful, if the systems are fully operational, or if the public announcement was exaggerated. This allows the US to accurately assess the true risk before making strategic decisions.

EPISODE 4.2 - B Is Guaranteed 📈

Moral hazard – Unobservable Effort

It's the first day of the semester. The professor of the infamous 'Weed-Out Science 101' — the class everyone dreads, walks in and says,

"Don't worry too much about grades. This course is curved. The bottom 10% get a C, and the class average will be a B, guaranteed. Let's see if you guys can learn better in a relaxed environment."

The room collectively exhales in relief. Including Leo.

Between rehearsals for his music club's upcoming overseas performance and his weekly guitar practices, Leo barely has time to breathe — let alone study 30 hours a week for this core science course.

He does some quick mental math:

'I just need to stay slightly above the bottom 10%. That's fine. I don't need to push for an A if I'll get a B anyway.'

And so, Leo adjusts his "strategy": Less studying. More guitar. More sleep.

As the semester rolls on, he stays comfortably in the middle of the pack… never excellent, never failing. He submits his lab reports late but polished enough to pass. He memorizes just enough for quizzes.

When finals come, he's calm. Too calm.

Weeks later, the results are out: Final Grade: B.

But as he packs away his notes, he glances at the textbook. Thick, untouched, and filled with concepts that he will need for his next semester.

He got the grade, sure… but not the understanding.

The professor's "safety net" worked and that's exactly the problem.

GAME MODELING

Moral Hazard

We have introduced the dilemma of information imbalance, *market failure*, caused by adverse selection: the buyer doesn't know the product's type *before* the transaction. Now we introduce the **post-contract issue**: **Moral Hazard**, which also leads to the same dilemma.

Mechanism of Moral Hazard: The market with asymmetric information causes **one party's actions being unobservable after the contract is signed**. This unobservable action creates a conflict of interest, leading directly to market inefficiency.

- **The Student's Effort (Leo's Case):** If a student (Leo) is protected by a guaranteed curve, this safety net discourages high effort. Since poor performance is protected, Leo chooses to reduce his effort without professor's recognition. This unobservable effort causes moral hazard.

- **Insurance Behavior:** Once a person has comprehensive insurance, the effort they put into avoiding risk becomes unobservable from insurer, letting these buyers take advantage of the insurance after they buy it (e.g. engaging in riskier health behaviors).

- **The Employee Market:** A company cannot fully observe the CEO's effort outside the office. The CEO's private incentive (to relax) does not align with the company's interest (to maximize productivity).

GAME TRANSFORMATION

How to Prevent the Moral Hazard?

The idea is to **align incentives** by making the agent **bear a portion of the risk** or by increasing the penalty for low effort.

- **Leo's Case Solution:** The professor must **tie the student's output directly to their pay-off**. Solutions include replacing the guaranteed curve with grading systems that require a high minimum absolute performance to pass, or implementing frequent, mandatory, and closely monitored assignments or peer review to make effort more observable.

- **Insurance Market Solution:**
    - **Cost Sharing:** The buyer must pay some amount (e.g. a deductible or co-payment) to be financially punished for unnecessary risks or claims, thus avoiding moral hazard.

- **Technology:** As technology offers powerful potential for transparency, solutions can be implemented to make information more transparent. For example, insurers can monitor buyer claim patterns more closely using advanced data analytics.

  However, implementing this technology requires significant upfront investment, which is a consideration that still needs careful balancing.

- **Employee Market Economic Solution:**
  - **Performance-Based Pay:** Setting regulations that reflect *put more input (effort), getting more output (wages).*
  - **Efficiency Wages:** Offering a wage high enough that the employee or CEO is scared to be fired because they know they cannot find this much high-salary job elsewhere.

If we integrate **these punishment and incentive mechanisms with the Signaling and Screening methods**, the market would move much closer and closer to the efficient outcome found in a complete information setting.

Application

Extra Real-World Example: Fixed-Price Home Renovation

When hiring a contractor for a "fixed-price" renovation, the contractor may have a tendency to use cheap materials or rush the job (Hidden Action). Since they have already secured a set amount of money, they might cut corners to save their own time and costs, knowing you bear the long-term cost of poor quality.

To solve this, you as employer, can switch to "Progress-based Payments." Instead of paying everything upfront, you release funds only after specific milestones are inspected and approved. Additionally, including a "Warranty Period," where the contractor must fix any issues for free, forces the agent to **bear a portion of the risk**. If the work is poor, they will lose their own time and resources to correct it, effectively aligning their incentives with your goal of high-quality work.

Season 5: Other Games

Since we mentioned in Season 1, game theory does not always have to use the Nash Equilibrium concept. This final season covers special, interesting games that solved by entirely different methods.

EPISODE 5.1 - Everybody's Field, Nobody's Responsibility 🌱

Externalities and Tragedy of the Commons

From casual hobbies to real football players, George and Leo have come a long way.

After joining Tom and his friends' football team, they have been playing every Friday and weekend on the university's sports field. Their teamwork and energy caught everyone's attention, and soon… they were selected to represent CCT University in an intercollegiate football match!

But just as their training was ramping up, a new challenge appeared…

Not from a rival university… but from the Arts Club.

One afternoon, George and Leo arrived at the field only to find one side fenced off — filled with half-finished sculptures and piles of *recycled art parts.*

"Is this… an art exhibition or a junkyard?" George whispered.

"Maybe both," Leo sighed.

The Arts Club had taken over part of the field to dry their clay sculptures for a week or two. They even left scraps and tools lying around. The messy corner not only ruined the view but also made others afraid of accidentally breaking something valuable.

A small sign on one sculpture read: "Please don't touch. Will remove in two weeks. No storage space available."

That didn't help.

To most students, it felt like the public field had turned into a private warehouse… or worse, a *garbage zone with creative justification.*

But that wasn't the only issue.

The football field — open to everyone at CCT University — had no clear schedule or usage rules.

Their Football Team wanted daily practice.

The Athletics Club wanted to run laps.

And other students just wanted to casually kick a ball after class.

Everyone had the same motivation: **Use the field as much as possible, the more we use it, the more our group benefits**.

But the cost… the muddy patches, the dead grass, the ruined turf was shared by everyone.

The result? **Overuse. No one cared about maintenance.**

Soon, the lush green grass turned patchy, then muddy. By the rainy season, the field had become a swamp. No one could play anymore… not footballers, not runners, not even the Arts Club.

Leo kicked a dried clump of mud and muttered, "So much for teamwork…"

George scratched his head. "Guess this is the real-life version of a **Tragedy of the Commons.** Everyone tries to win, but in the end, everyone loses."

Tom, sitting nearby while tying his shoes, grinned. "Maybe we should apply some game theory to fix this field before it disappears?"

Because they weren't the type to give up easily. The next day, George, Leo, and their team planned to meet with the student council to propose a fair, cooperative, and sustainable schedule for everyone.

After all, if there's one thing they've learned… *Some games are only fun when everyone wins.*

GAME MODELING

Externality

An **Externality** is a cost or benefit that falls upon a third party who was not directly involved in an economic transaction and is not compensated for the impact. It is an unintended side effect of one person or one group's action.

- **Negative Externality:** In the grass field scenario, the Arts Club's decision to use the field for storage created a negative externality for other students (footballers, runners). It imposed a cost (loss of usable space, ruined aesthetics) the Arts Club compensating them.

Externalities lead to non-optimal allocation because the actor (e.g. the Arts Club) does not bear the full social cost of their action. Since they have no need to pay the full cost, we could say the actor gain **profits**. The Arts Club will continue using the field to dry their clay sculptures because they do not have to pay for the resulting damage or inconvenience to others.

Tragedy of the Commons

The **Tragedy of the Commons** is an economic problem where a resource open to all, is inevitably overused and depleted due to individuals acting in their own self-interest.

This problem occurs because the resource has two critical characteristics:

1. **Non-Excludable:** everyone can access it. It is impossible or too costly to exclude people from using the resource (e.g. the university cannot easily block every student from the field).
2. **Rivalrous:** One person's use of the resource diminishes another person's ability to use it (e.g. the Football Team's intense practice wears down the grass, reducing the quality of the field for the Runners).

Every user has an incentive to use the resource as much as possible, while the cost of degradation (mud, dead grass) is distributed among everyone, ultimately leading to the resource's destruction.

Two Main Issues Causing Inefficiency

In our journey through game theory, the two main categories that cause resource allocation inefficiency in a market:

1. **Asymmetric Information – Dilemmas of Information: Caused by private information** (e.g. a seller knowing more than a buyer). This to a lack of buyer confidence, causing **market failure** (like the Market for Lemons) where the transaction price does not accurately reflect the product's true value. This type of dilemma is solved through **Signaling** to reduce information asymmetry.

2. **Externalities and Tragedy of the Commons – Dilemmas of Property: Caused by undefined property rights,** leading to inefficient resource allocation outcomes and overuse of common resources, because a player's decision only considers their private cost and benefit, ignoring the full social cost or benefit.

The Root of the Problem: Lack of Clear Property Rights

The core issue underlying both externalities and the Tragedy of the Commons is the **lack of clear property rights.**

- **Externalities:** the problem here is that the right to use the environment or space is unclear (Who can use the lawn without interruption?). The **Coase Theorem** solves this by establishing a clear right and enabling negotiation.

- **The Tragedy of the Commons:** this is a severe form of externality where the non-existence of property rights (like having an open sports field for students, or "*public goods*") encourages everyone to overuse the resource until it is destroyed. This means a complete depletion of resources due to individual self-interest. To solve this, we have to establish **clear Property Rights (Privatization) or Regulate usage to limit it**.

GAME TRANSFORMATION

The Solution: Clear Property Rights

The solution to these property-related dilemmas lies in defining and enforcing clear property rights, or *ownership*, to make users internalize the public costs.

1. **Private Solution to solve externalities: The Coase Theorem**

   The core idea is, if **property rights** are clearly defined and transaction costs are zero, private "negotiation" between affected parties (e.g. the Arts Club, Football Team, Athletic Runners Club, and representative of common students) will always lead to a **socially optimal allocation**, regardless of which party is initially granted the rights.

   This theorem shows that the problem is not about who is "right," but rather that the right to use the resource is not clearly established. Once the right is established, the market works to find the most efficient allocation through bargaining.

   Application to the Field Scenario: If the Football Team is granted the exclusive right to a clean field, the Arts Club must pay them to "buy" the right to display their work. Conversely, if the Arts Club is granted storage rights, the Football Team must pay them to move their sculptures. The clay sculptures will remain only if the benefit to the Arts Club is truly greater than the aesthetic damage and inconvenience the other students have to bear.

2. **Public Solutions to solve the Tragedy of the Commons: Privatization and Regulation**

   To prevent the collapse of shared resources, society must establish clear property rights or introduce specific limits.

   A. **Privatization:** Assigning **clear property rights** (e.g. to the Campus Recreation Department) forces users to **internalize the cost** of their actions through fees or fines for damage: The Football Team would have to pay a scheduling fee to book a slot, and the Arts Club would have to pay a storage fee. Since real users now bear the full cost, they will only use the field when the benefit outweighs the expense, preventing overuse.

This approach mirrors the logic of the **Matching Market** (Episode 3.4). In that model, the dilemma of multiple buyers competing for a single item is resolved by increasing the price. Similarly, privatization solves the "Common Resource Dilemma" by using **Price as a Tool to Solve Conflict**. The item or space is automatically allocated to the party that willing to pay the *Seller* (the owner of the resource) to use the property.

B. **Regulation to Limit Usage:** Implementing strict rules, such as limited sport practice hours, banning heavy items like sculptures from the grass, requiring all users to contribute to maintenance or face suspension of field privileges, restricts the individual's incentive to overuse the resource for the greater good of the community.

Both solutions achieve the goal of aligning individual incentives with the collective interest, moving away from the unsustainable **free-for-all** to Win-Win outcome, preventing the widespread destruction inherent in the **tragedy of the Commons**.

Application

Positive Externalities

While most discussions focus on negative externalities (like visual pollution in the sport field scenario), some externalities can benefit society:

1. **Knowledge Spillovers:** A streamer or educator sharing content benefits their immediate audience, but the knowledge shared also positively impacts viewers outside of the paid subscriber base, improving general knowledge.

2. **Encouraging Action:** When one family member travels, the experience and stories they share with the rest of the family can encourage other family members to travel or explore new things, generating a **positive external benefit** within the family unit.

EPISODE 5.2 - Cut or Wait? ⏳

Evolutionary Game

Lately, George and Sara, now officially a couple, have been spending their lunch breaks together at the campus dining hall. They'd sip afternoon coffee, chat playfully, and share a small dessert before heading to class.

After coming here so often, they've started to notice a pattern in people's behavior.

From watching others in line, there seem to be two types of strategies. Some people choose to **be aggressive and try to cut in line** to get ahead. Others prefer to **be polite and wait patiently** for their turn.

Now, as George and Sara join the long line for dessert themselves, they can't help but wonder…

Which strategy actually gets you the higher payoff?

GAME MODELING

Cut or Wait?

This scenario introduces a new dimension to game theory by applying the concept of **Evolutionary Game Theory** to a common social dilemma: standing in line. George and Sara must choose between two common strategies (or behaviors) while waiting in a busy line:

- Strategy X = Be aggressive, trying to cut in line
- Strategy Y = Be polite, waiting patiently in line

		The girl behind them	
		X : Cut in line (1-p)	Y : Wait in line (p)
George & Sara	X : Cut in line (1-p)	(0, 0)	(4, 3)
	Y : Wait in line (p)	(3, 4)	**(6, 6)**

Figure 5.2.1 Cut or Wait?

The possible outcomes:

- If both choose **Strategy Y (Wait)**, the line moves smoothly, avoiding conflict and stress, resulting in the highest mutual payoff of **(6, 6)**.
- If one is polite (**Y**) and the other cuts (**X**), the cutter gains an immediate benefit (4), while the polite individual incurs a minor delay (3).
- If both choose **Strategy X (Cut)**, they clash, attracting negative attention from staff, leading to the worst mutual payoff of **(0, 0)**.

The Search for a Dominant Strategy:

We begin by applying standard game theory analysis to find the optimal strategy for the two parties involved in the immediate interaction.

The girl's perspective:

- If George & Sara chooses X, then the girl chooses Y as 3 > 0.
- If George & Sara chooses Y, then the girl chooses Y as 6 > 4.

The Girl has a single **Dominant Strategy**: **Strategy Y (Wait)**.

George & Sara's perspective:

- If the girl chooses X, then George & Sara chooses Y as 3 > 0.
- If the girl chooses Y, then George & Sara chooses Y as 6 > 4.

George & Sara also have a single **Dominant Strategy**: **Strategy Y (Wait)**.

Nash Equilibrium

Since both players possess a dominant strategy of **Y (Wait)**, the single **Pure Strategy Nash Equilibrium** for the one-time interaction is **(Y, Y) (6, 6)**. While the game has a standard pure strategy Nash Equilibrium, the primary goal is to use the framework of population dynamics to determine the *long-term stability* of the courteous strategy.

Evolutionary Game Analysis

Evolutionary Game Theory explains what a large population *will* do over time. We assume that successful strategies (the one with higher payoffs) are replicated more often over time.

- **Start with strategy X**

To find the **Evolutionarily Stable Strategy (ESS)**, we compare both strategies' payoffs when **the most of the population is dominated by Strategy X**.

- Let "p" be a *small* proportion of people playing Strategy Y (Wait)
- Let "1-p" be **the much larger** proportion of people playing **X (Cut)**

Expected Payoff for X: $\underline{\mathbf{0}}$*(1–p) + 3*p = 3p

Expected Payoff for Y: $\underline{\mathbf{4}}$*(1–p) + 6*p = 4+2p

Since p is very small (≈ 0), we only compare the starting expected payoffs: $\underline{\mathbf{0 < 4}}$. This means, Strategy X gives less payoff than Strategy Y. Therefore, **X is NOT an evolutionarily stable strategy**. On the other hand, Y is an ESS.

In conclusion, over time, individuals using strategy X will decrease from seeing their strategy yield lower average returns. The entire population shifts to using strategy Y because Strategy Y is the more stable choice in the long term.

At the end, everyone using the dining hall will be polite and wait patiently in line to get their turn, proving that courtesy is the superior, stabilizing behavior in this dynamic.

General Matrix of Evolutionary Game

Evolutionary Game is typically applied to **symmetric information games**, where the players choose from two strategies, S and T.

		Population B	
		S (1-x)	T (x)
Population A	S	**(a, a)**	(b, c)
	T	(c, b)	(d, d)

Figure 5.2.2 Evolutionary game matrix

x is the proportion of the population playing Strategy T.

1-x is the proportion of the population playing Strategy S.

Expected Payoffs:

Expected payoff for S: a(1-x)+bx

Expected payoff for T: c(1-x)+dx

Strategy S is Evolutionarily Stable precisely when either:

1. $\boxed{a > c}$ (S is a better response against S than T is), or
2. $\boxed{a = c}$ AND $\boxed{b > d}$ (S is equally good against S, but better against T).

Evolutionary Strategy and Nash Equilibrium

An **Evolutionarily Stable Strategy will always be a Nash Equilibrium** because the ESS condition requires the strategy to yield the best payoff against itself, which is the definition of a stable pure-strategy Nash Equilibrium.

However, **not all Nash Equilibrium strategies are Evolutionarily Stable Strategies.** This occurs when a Nash Equilibrium is *weakly dominated* by another strategy.

		Player B	
		X	Y
Player A	X	**(1, 1) (a, a)**	(2, 1) (b, c)
	Y	(1, 2) (c, b)	**(3, 3) (d, d)**

Figure 5.2.3 When a strategy is weakly dominated, it is not an ESS

In this matrix, Strategy **X is a Nash Equilibrium**. However, it is a **weakly dominated strategy** because Y gives a payoff equal to or better than X in every situation.

Applying the ESS conditions to **test if X is stable against invasion by Y**:

1. Is $\boxed{a > c}$? : No, 1 is not >1.
2. Is $\boxed{a = c}$ AND $\boxed{b > d}$? : a = c is true (1 = 1). However, b > d is not true because 2 is not > 3.

Since neither principle can be true, **X is NOT an Evolutionarily Stable Strategy**, even though it is a Nash Equilibrium. While X may be acceptable in the short run, the superior outcome (Y,Y) = (3,3) will **dominate** X in the long run because Y has better long-term fitness in the environment.

Game Theory with Biology

This game solution applies the Nash concept and mixes it with **evolutionary biology.** The main idea of Evolutionary Game Theory is that **the fitness of an organism, or strategy** in a population is determined by the **expected payoff it receives** from an interaction with a random member of the population.

Strategies, which are the observed characteristics (like Strategy X and Y in George and Sara's queueing scenario), that yield **higher fitness** are replicated more often through a process analogous to

natural selection. At the end, the entire population shifts to using that superior strategy because it is the more stable choice in the long term.

This theory is defined by its stability against invasion:

- **Invasion:** A new strategy **T invades** a population playing **S** at a *small* level x, meaning an x fraction uses T and a 1–x fraction uses S.
- **Evolutionary Stability:** Strategy **S is evolutionarily stable** if there is a positive number y such that when the other strategy **T invades S** at any level x < y, **the fitness (payoff) of the population playing S is strictly greater than the fitness of the population playing T**. This means the "mutant" strategy T will be eliminated over time.

Coexistence and the Impact of Conflict Costs

The original evolutionary game is the Hawk-Dove Game. It models the stable mix of aggressive and passive behavior in a population competing for resources:

		Beetle B	
		Hawk	Dove
Beetle A	Hawk	(-2, -2)	**(4, 0)**
	Dove	**(0, 4)**	(2, 2)

Figure 5.2.3 Beetle A and B are competing for a resource (like food or territory)

The possible outcomes:

- If both choose Dove, they share the resource without conflict. Both get half the value (2, 2).
- If one plays Hawk and the other plays Dove, the Hawk takes the full value (4), and the Dove gets nothing (0).
- If both choose Hawk, they fight, resulting in incurring the cost of injury (-2, -2).

The pure-strategy Nash Equilibrium are (Hawk, Dove) and (Dove, Hawk). However, this game is fundamentally solved by its Evolutionarily Stable Strategy, which must be a **mixed strategy**. The population stabilizes when the proportion of Hawks (p) and Doves (1–p) reaches a point where **the expected payoff of playing Hawk equals the expected payoff of playing Dove**. The ESS is a mixed population of both behaviors.

The result of a mixed population of behaviors shows that the ESS not only reflects how two kinds of characteristics, aggressive and peaceful, fit with the environment, but also explains how they **coexist**.

As everyone wants to be the Hawk to take advantage of others, but if two Hawks meet, it causes a dilemma. **This stable mix is driven by the potential cost of that dilemma**:

- **Low Conflict Cost:** If the damage of two Hawks meeting is *not high enough* (e.g. payoff is (1, 1)), the cost is easily tolerated, and the population would favor the aggressive Hawk strategy until it becomes dominant.

		Beetle B	
		Hawk	Dove
Beetle A	Hawk	*(1, 1)*	**(4, 0)**
	Dove	**(0, 4)**	(2, 2)

Figure 5.2.4 Low Conflict Cost

- **High Conflict Cost:** If the damage of two Hawks is **high enough** (e.g. (-3, -3)), it forces individuals to turn toward the safer Dove behavior, allowing the mixed population to stabilize. The ESS emerges precisely because the extreme outcome is punished severely.

		Beetle B	
		Hawk	Dove
Beetle A	Hawk	*(-5, -5)*	**(4, 0)**
	Dove	**(0, 4)**	(2, 2)

Figure 5.2.5 High Conflict Cost

The ESS ensures that the two competing characteristics (predator and prey, powerful and weak) coexist, because the cost of over-specialization or unchecked aggression is too high, forcing a dynamic balance.

GAME TRANSFORMATION

How Weakness Survives?

A common dilemma in evolutionary games is: **certain low-level players may never have the opportunity to ascend to a high-level status**. Within the Hawk-Dove framework, a Dove existing in a world dominated by Hawks may never win a primary resource, only survive on the leftovers. However, the low-level strategy does not disappear. This reflects a **Biological Principle**.

A Dove strategy persists not by winning, but by avoiding the serious costs associated with Hawk-Hawk conflict. By settling for a lower, "safe" payoff, the Dove ensures its species continues, while the Hawks risk total elimination through mutual injury.

This creates a **critical balance** within the ecosystem. If every low-level player attempted to become a high-level player (every Dove became a Hawk), the total fitness of the population would crash due to conflict costs. No one would survive long enough to replicate.

Thus, we see a state of **forced coexistence**. The low-level strategy survives because **it is necessary for the stability of the entire system**. In nature, this is mirrored in the relationship between predator and prey, such as the lion and the deer. If the high-level predator becomes too successful and consumes all of its low-level prey, the predator itself faces extinction as there is no food left. After the lion has gone extinct, the deer population would become numerous and aggressive, eventually running out of grass, leading to a situation where everyone faces extinction and the ecosystem crashes.

In conclusion, the ESS explains why diversity exists as a requirement for life. **High-level and low-level players are locked in a coexistence**, where the cost of everyone being "high-level" is a total extinction of the species.

Application

Real-World Example: Tech Giants and Startups

In the global technology market, we often see a forced coexistence between massive, High-Level companies and small, Low-Level startups. A tech giant possesses the resources to acquire or crush any small competitor that enters its space. However, if the giant becomes too aggressive and eliminates every startup, the ecosystem of innovation dries up. Without the Low-Level startups to act as a laboratory for new, risky ideas, the giant becomes stagnant and over-specialized. Eventually, a shift in the global market occurs that the giant is too slow to adapt to, leading to its eventual downfall.

The Low-Level players survive because they provide the R&D that the giant needs to stay relevant. The giant allows the startups to exist because a market with zero competition eventually loses its ability to evolve, leading to a total industry crash.

EPISODE 5.3 - This Was Supposed to Be Easy 🎬

Group decision – Voting

Ever since George got together with Sara, Leo has been feeling a little... left out.

So lately, he's started chatting with Mia — Sara's close friend.

After weeks of texting and late-night jokes, Leo finally asks her out for a movie date. But it's their first movie together, and Mia's a bit nervous.

"Let's invite Sara too," she suggests. "We can use that 'Buy 2, Get 1 Free' movie ticket promo."

So now, it's the three of them, **Leo, Mia, and Sara**, planning one movie night.

There's just one small problem: choosing what to watch.

They all have different tastes. Still, they must agree on one movie for their *group date*. The options are:

A: A thrilling Action Movie that Leo loves.

J: A Japanese Animation that Sara finds fascinating.

C: A Classic Drama that Mia has wanted to see for months.

Three friends. Three preferences.

Only one movie can win.

It's time for… *the Voting Game.*


ACTION MOVIE
JAPANESE ANIMATION
CLASSIC DRAMA

GAME MODELING

Movie Night Decision : Majority Rule

Leo, Mia and **Sara** are deciding which film to watch together this weekend. They must choose one option for the group:

- **A:** A popular Action Movie
- **J:** A Japanese Animation
- **D:** A Classic Drama

The binding decision must reflect the collective preference, but their individual rankings are in conflict:

Voter	1st Choice	2nd Choice	3rd Choice
Leo	A (Action)	J (Animation)	D (Drama)
Mia	J (Animation)	D (Drama)	A (Action)
Sara	D (Drama)	A (Action)	J (Animation)

Figure 5.3.1 Individual's preferences

Let's test the choices of Leo, Mia and Sara using the first voting method, **Majority rule** (pairwise voting):

1. **A vs J** (Action vs Japanese Animation): Leo and Sara prefer A. Mia prefers J. So, **A wins over J** with 2:1 majority.
2. **J vs D** (Japanese Animation vs Drama): Leo and Mia prefer J. Sara prefers D. So, **J wins over D** with 2:1 majority.
3. **D vs A** (Drama vs Action): Mia and Sara prefer D. Leo prefers A.
   So, **D wins over A** with 2:1 majority.

Although individual's preferences are transitive, no cycling, the voting result can show a **Cyclical Preference:** A → J → D → A. This is known as the **Condorcet Paradox.**

Since the group prefers A to J, J to D, but D to A, the choice is stuck in a cycle. This means the group as a whole *cannot* reach a rational or stable decision. The movie selection game has no equilibrium winner under simple majority rule.

Movie Night Decision : Positional Voting (Borda Count)

Since the Majority Rule method cannot produce a single rational group ranking, we use **Positional Voting** to create a stable group preference directly from each player ranking.

The specific method of positional voting we will use is the **Borda Count**, which assigns points based on rank. The choice with the highest total score wins.

True Preferences (A vs J Only):

Let's assume D – Drama's movie tickets are already sold out. So, the fight now is actually just between A (Action) and J (Japanese Animation).

We assign points based on the number of options (N = 2). If a choice is ranked:

- **1st Choice (Highest Rank):** N-1 = 2-1 = 1 points
- **2nd Choice (Middle Rank):** N-2 = 2-2 = 0 points

Voter	1st Choice (2 pts)	2nd Choice (1 pts)
Leo	A	J
Mia	J	A
Sara	A	J

Figure 5.3.2 Removing D to determine whether A or J is more preferred

Choice	Leo	Mia	Sara	Total Score	Group Rank
A	1	0	1	2	**1st**
J	0	1	0	1	**2nd**

Figure 5.3.3 True preference after removing D

True Preference Conclusion: A – Action is the clear group winner with 2 points.

Strategic Voting with a "Spoiler" (A, J, and H):

Now, the truly irrelevant choice **H (Horror)** is added, bringing the total choices to N=3. If a choice is ranked:

- **1st Choice:** N-1 = 3-1 = 2 points
- **2nd Choice:** N-2 = 3-2 = 1 point
- **3rd Choice:** N-3 = 3-3 = 0 points

Voters who want their movie to win must strategically rank **H** in the **middle slot (2nd)** to push their main opponent into the **lowest-scoring slot (3rd)**.

Let's assume the voters are now voting strategically:

Voter	1st Choice (2 pts)	2nd Choice (1 pt)	3rd Choice (0 pts)	Strategic Goal
Leo (Wants A)	A	H	J	Leo is happy with A winning.
Mia (Wants J)	J	H	A	**MIA IS STRATEGIC:** Uses H to push her rival (A) to the 3rd rank (0 points).
Sara (Wants A)	A	H	J	**SARA IS STRATEGIC:** Uses H to push her rival (J) to the 3rd rank (0 points).

Figure 5.3.4 Voters may benefit by lying their preferences .

We calculate the total score for each movie based on the three manipulated rankings:

Choice	Leo's Rank	Mia's Rank	Sara's Rank	Total Score	Group Rank
A	2 pts	0 pts	2 pt	4	1st
J	0 pt	2 pts	0 pts	2	3rd
H	1 pts	1 pt	1 pts	3	2nd

Figure 5.3.5 The misreporting result of the top-ranked choice

Manipulated Preference Conclusion:

- **A (Action)** wins with 4 points.
- **H (Horror)** (the irrelevant choice) comes in 2nd place with 3 points.
- **J (Animation)** is pushed into last place with only 2 points.

By ranking the truly irrelevant option (H) in the middle, they push their main competitor into the lowest score category, demonstrating how strategic voting and the presence of a "spoiler" can completely change the outcome and manipulate the group's true ranking.

While Positional Voting avoids the paradox of the Majority Rule, it is highly susceptible to manipulation through strategic voting and the inclusion of alternatives that are only present to downgrade a competitor's total score.

Why Voting?

Voting is a crucial mechanism, used to **aggregate individual beliefs** and **synthesize information** into a single, binding **group decision**. While markets aggregate private information through prices, voting explicitly synthesizes personal preferences to reach one collective choice that speaks for the entire group. Thus, voting has the power to manipulate or move a society because everyone has participated.

There are many voting methods, and each method has a significant effect on both the process and result for reaching a group decision. It also can be used in choosing a single collective winner or producing a ranked list of alternatives.

Majority Rule

The **Majority Rule** is the simplest and most common voting system. It takes a collection of complete and transitive individual preference relations and attempts to produce a group ranking. For easier discussion, we will often assume that the number of voters in a scenario is odd.

The process of Majority Rule relies on **pairwise voting**: comparing every single alternative to every other alternative (e.g. A vs J, J vs D, A vs D) and selecting the winner based on which option receives more than half the votes.

The Condorcet Paradox

In a voting system that uses **Majority Rule** (simple pairwise voting), a major issue arises when there are three or more alternatives. Sometimes when we pit the options against each other one pair at a time, we encounter a **Condorcet Paradox**, which is a cyclical preference like the A → J → D → A cycle in the movie selection scenario.

This paradox demonstrates that the majority preferences of the group can be **cyclical**, leading to an outcome where no single choice can be considered the overall winner. This proves that Majority Rule, while simple, is not a reliable method for aggregating complex preferences.

Positional Voting and Borda Count

There is another voting method that can avoid the Condorcet Paradox. **Positional Voting** is a method that determines a group ranking by assigning a specific number of points (a weight) to each rank position on a voter's ballot.

A specific type of positional voting is the **Borda Count**, which calculates the total number of points each alternative receives across all ballots. The final choices are then ordered according to their total weight. This method is often used in settings where a ranked list is valued, such as sports competitions or academic rankings, because it considers the full depth of a voter's preferences, not just their first choice.

Issues in Positional Voting

While positional voting avoids the cyclical failures of the Condorcet Paradox, it introduces a new set of vulnerabilities.

A major weakness is that adding an **irrelevant choice can act as a spoiler** to downgrade the overall score of a top-ranked competitor. This is known as **third-party effects**. Because of this, voters can sometimes benefit by **lying** about their true preference, for instance, by strategically ranking a less-preferred option higher to push their competitor's score down. This manipulation influences the misreporting result of the top-ranked choice and violates the principle of **Incentive Compatibility**, where voters are incentivized to report their true preferences.

Arrow's Impossibility Theorem

The systemic failure of **Majority Rule** (the Condorcet Paradox) and the manipulation risk in **Positional Voting** (misreporting and spoiler effects) are formally proven by one of the most powerful conclusions in social choice: **Arrow's Impossibility Theorem**.

This theorem proves that it is **impossible** to design a group decision rule or a voting system that aggregates individual preferences into one collective, complete group ranking while simultaneously satisfying three basic criteria:

- **Independence of Irrelevant Alternatives (IIA):** The group's preference between two choices (say, A and B) should *only* depend on how individuals rank A versus B, not on how they rank some third, irrelevant option (C).
- **Pareto Principle:** If every single voter prefers choice A over choice B, then the group ranking *must* also rank A over B. This is the minimal standard of rationality for a group decision.
- **Universal Domain:** The system must be able to handle *any* possible combination of individual preferences (even illogical or cyclical ones).

Arrow's theorem concludes that any system satisfying these criteria will inevitably lead to **inconsistent and cyclical group preferences** when there are three or more choices. This stark result suggests that an ideal voting system cannot exist.

The Voting Dilemma

Arrow's theorem reveals a deep **dilemma** in group decision-making: the *only* system to produce a ranked list that satisfies all of the properties in the theorem is a **Communist** system (Dictatorship), where the group ranking is simply equal to the ranking provided by a single player *i*.

This means a perfectly rational, fair, and stable collective choice mechanism (like democracy by Majority Rule or Positional Voting) is **mathematically impossible**. The conflict between the ideal of a fair outcome and the mathematical reality of its impossibility creates a dilemma for democracy:

1. **Dilemma of Imperfect Democracy:** Since it is difficult to find a set of alternatives that satisfies everyone's preferences, the imperfect result causes a dilemma. We know that voting results can be **manipulated** or cheated by some players (e.g. through strategic misreporting or "spoiler" choices), meaning the outcome is not always **Incentive Compatible**. This shows that democracy, while superior to communist system, is still an imperfect system susceptible to game-playing.
2. **Dilemma of Unfair Communist:** The alternative, switching to a communist system for the sake of stability, causes a greater dilemma, as the entire system becomes fundamentally **unfair**.

The chaos of cyclical preferences in democracy is, therefore, the price we pay for avoiding the unfairness of a communist system. This forces us to focus on practical solutions, such as the structural constraint of **Single-Peaked Preferences**, rather than seeking a perfect, generalized voting system

GAME TRANSFORMATION

How to Solve the Voting Dilemma?

As a perfect voting system is impossible, the practical solution is to sacrifice one or more of Arrow's principles, or to change the structure of the alternatives themselves. To ensure a stable outcome, the preferences of the voters must often satisfy a special condition known as **Single-Peaked Preferences**.

This condition applies when all alternatives can be meaningfully arranged along a single, one-dimensional line (e.g. from least expensive to most expensive, or from liberal to conservative). **Each voter has one ideal point** on this line where their satisfaction is maximized. Their preference for any other alternative decreases steadily as those alternative moves farther away from their ideal point.

- **The Winner:** When all voters have single-peaked preferences, the Condorcet Paradox (A → J → D → A) **cannot arise**. The stable group decision will always be the option preferred by the **Median Voter** (the voter whose ideal point is exactly in the middle of the group).

Therefore, in games involving group decisions, achieving a fair and stable outcome depends not just on finding a Nash Equilibrium, but on how the available alternatives are structured to align with this single-peaked condition.

Application

Real-World Example: Corporate Board Elections

Most major corporations, such as those listed on the S&P 500, traditionally use a **Plurality Voting System** for their Board of Directors. In this narrative, shareholders have a "Take it or Leave it" choice. Each seat is contested, and the candidate with the most "For" votes wins, regardless of whether they reached a majority.

However, to prevent a "Communist-style" absolute control by a few large owners, many have shifted to **Majority Voting**. In this system, a director must receive more than 50% of the votes to be seated. If they do not, they must resign. This forces the winning candidate to satisfy the **Median Voter** (the middle-ground shareholder), ensuring the leadership has broader support and the system remains stable.

EPISODE 5.4 - Wait… That's Not 50-50 ⚖️

Outside Option and Nash Bargaining Solution

Today, UberEats is celebrating its anniversary with a special promotion… **order together with someone and get $100 off!**

George and Leo's phones buzz at the same time.

"Wait… $100 discount?!" George exclaims.

"No way! That's huge," Leo grins.

Now, both George and Leo are planning to order something for dinner. But they each already have a potential partner — with… not-so-equal offers.

Sara messages George:

"Hey honey, want to pair up for the promo? I'll take $70 of the discount. You can keep $30."

Meanwhile, Notification from Tom pop up on Leo's screen:

"Bro! Let's order together! But… I want $55 of the discount. You can take $45."

George and Leo both pause, glance at their phones… and at each other.

The question is:

Should they go along with their friend(or lover)s' offers, or make a deal directly with each other?

The bargaining game begins.


UberEats Anniversary! $100 Off
Sara
Hey honey, want to pair up for the promo? I'll take $70 of the discount. You keep $30.
Tom
Bro! Let's order together! But… I want $55 of the discount. You can take $45.

GAME MODELING

Four Nodes

If George and Leo decide to order together, how would they bargain to split the promotion fairly?

Let's first establish their **Outside Options**:

- **George** can get **$30** if he orders with Sara.
- **Leo** can get **$45** if he orders with Tom.

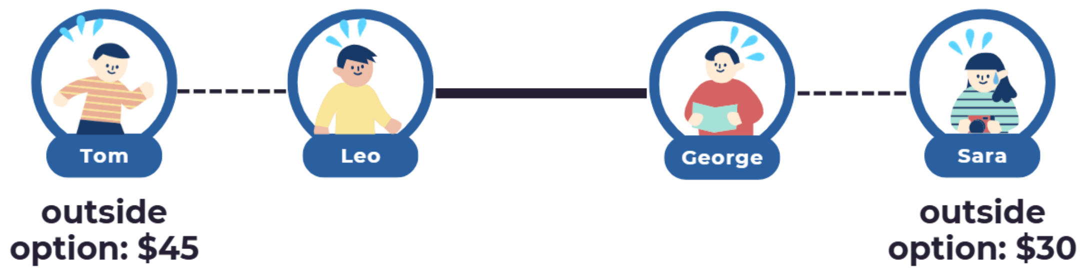


Figure 5.4.1 Outside Options and Mutual Surplus Analysis

Then, if George and Leo order together, the total discount is $100. To calculate the mutual surplus, the extra benefit they create by cooperating:

Mutual Surplus = Total surplus – (Outside options of George and Leo)

Mutual surplus is: 100 - (30 + 45) = 25.

According to the **Nash Bargaining Solution**, they can split the mutual surplus evenly:

For George, 30 + (25 / 2) = 42.5, which is his outside option plus half of the mutual surplus.

For Leo, 45 + (25 / 2) = 57.5, which is his outside option plus half of the mutual surplus.

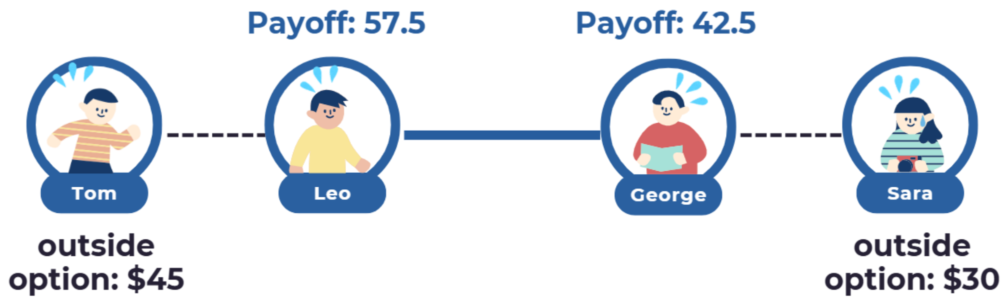


Figure 5.4.2 Nash Bargaining Solution

Five Nodes

Now, their mutual friend **Mia** also wants to order together, making Sara and Tom change their offers.

For George, he has two outside options, Mia and Sara, who would give him $45 and $50 discounts respectively. Since only two people can order together to get the promotion, George may choose to order with **Sara ($50)** as his second choice. He can get much more discount from Sara than with Mia.

For Leo, he also has two outside options, Tom and Mia, who offer $30 and $45 discounts respectively. He may choose to order with **Mia ($45)** as his second choice, since that gives him a higher discount than with Tom.

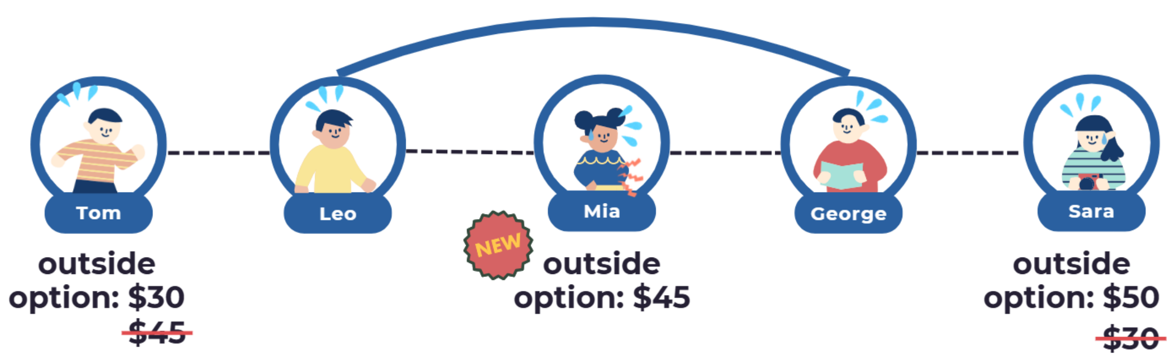


Figure 5.4.3 Network after adding a new node : Leo will use Mia (higher offer) as his outside option to bargain with George

So, George's outside option is **$50**, while Leo's is **$45**.

The mutual surplus: 100 - 50 - 45 = 5

According to the **Nash Bargaining Solution**, they can split the mutual surplus evenly:

For George: 50 + (5 / 2) = 52.5

For Leo: 45 + (5 / 2) = 47.5

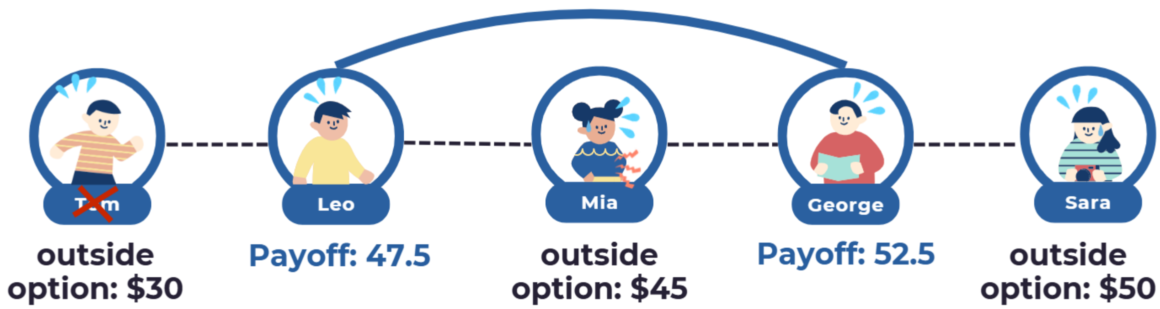


Figure 5.4.4 A new Nash Bargaining Solution

The outcome demonstrates that the player with the higher **outside option** has greater bargaining power and secures a larger portion of the final surplus, even though they split the **mutual gain** equally. This is a powerful demonstration that power in negotiation is directly related to your ability to walk away and still secure a high payoff.

Outside Option

Outside Option (or Reservation Utility) is the guaranteed payoff a player can secure **if the current negotiation fails.** It defines the minimum amount a player must receive to agree to any deal.

Nash Bargaining Solution

The **Nash Bargaining Solution** is a formal way to predict how two rational players or nodes will split a potential surplus, especially when their **outside options** differ. This model is critical because it prioritizes maximizing the product of the players' utility gains, rather than just the sum.

Structural Power

In network theory, the **power of a node** is not solely determined by its wealth or preferences, but critically depends on **its position** within the overall structure of the network.

In Nash Bargaining Solution, "**Bargaining power**" is formally defined as a node's **ability to limit the options of its trading partners** or its **ease of switching to an alternative partner**. This structural advantage is mathematically quantified by the value of a player's **Outside Option**.

The Source of Power: Lack of Alternatives

The key to having bargaining power lies in how **dependent** our trading partner is on us, specifically, whether they have viable **Outside Options** if we walk away from the deal. Power is therefore defined by the *lack* of alternatives for your partner.

- **Equal power nodes: X and Y**
    - In symmetric structures, all nodes have identical access to alternatives, leading to an equal **50:50** split of the surplus.
    - Outside Option: All outside options are equal, leading to equal bargaining power.

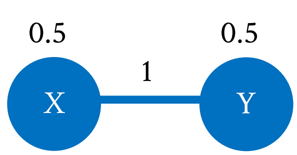


Figure 5.4.5 Mutual Surplus = 1 - 0 = 1, spitting evenly 1/2 = 0.5

- **High Power Nodes: B**
    - These are nodes that have **diverse alternative trading options** or act as **brokers** connecting otherwise separated groups. These nodes can credibly threaten their current partner (A and C): *"If you don't agree to my terms, I'll immediately trade with someone else."* This enables them to capture the largest share of the surplus.
    - **High Outside Option** grants them the leverage to capture the largest share of the surplus.

        Assume B trades with A, while *C offers B 0.8.*

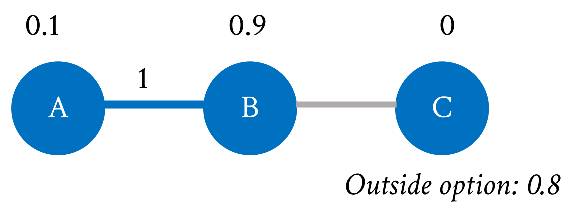


Figure 5.4.6 Mutual Surplus = 1 – 0 (A’s) – *0.8* (B’s) = 0.2, spitting evenly 0.2/2 = 0.1

- **Low Power Nodes (Peripheral Nodes): A and C**
    - These nodes face **high dependence** due to few or only one trading partner. They have no bargaining power because their partner can always credibly threaten to walk away. Consequently, these nodes receive the smallest share of the surplus, 0.1 and 0.
    - **Low or zero Outside Option** forces them to accept minimal offers.

Although we might initially feel that it is unfair that high-power nodes (like B in a path network) gain a higher payoff than low-power nodes (like A and C), in this context, fairness **does not mean receiving an equal split** of the total pie. Instead, it means the final outcome must compensate both players for their **Outside Option** (their minimum requirement), plus an equal share of the **Mutual Gain**. So, the player with the higher outside option receives a mathematically justifiable larger portion of the total surplus.

Exercising Force in the Bargaining Game

The mechanism of structural power is best explained through the simplest negotiation game: the **Ultimatum Game**. This game shows how the power imbalance of the network can be used to stabilize an unequal split.

- **Mechanism:** One player, the **powerful node** (Proposer), suggests how to divide a fixed sum of money with another, the **dependent node** (Receiver). The dependent node must either: **accept** the offer (and both get paid) or **reject** it (and both receive zero).

  Theoretically, the dependent node should accept any offer greater than zero (> 0) to maximize their benefit (since their outside option is 0). However, in practice, people often reject highly unfair offers (like a 99:1 split) to punish the powerful node's greed.

When applied to a network, the powerful node acts as the Proposer, making an unfair split. The less powerful node, lacking alternatives (a low Outside Option), is **forced to accept the unfair split** because the cost of rejection (receiving zero payoff) is too high. Thus, the position in the network dictates its ability to limit others' choices, establishing node's bargaining power.

Cooperative Games – No More Dilemma

The scenarios we analyze using the Nash Bargaining Solution exist within a **Cooperative Game** framework. This is a crucial shift from the Prisoner's Dilemma, as we now assume players are seeking a **joint outcome that maximizes their mutual payoff**, though they still have to bargain over the split.

The key characteristic of a cooperative game is the **elimination of the dilemma itself**, as the players are incentivized to achieve a joint, efficient outcome. However, A negotiation can only happen when players are in the **Bargaining Zone**, neither too close nor too far apart:

- **Not Too Close:** If the players are too close (e.g. family with *fully aligned* interests), the "game" disappears because there is no need to bargain. The result is trivial.

- **Not Too Far:** If they are too far apart (e.g. strangers), they will not even enter negotiations and might fall back into a Prisoner's Dilemma-like outcome where no deal is struck.

In the Bargaining Zone, players **trust each other enough to negotiate** but still lack the full, perfect alignment of interest where the game would disappear. This is why the bargaining game is often seen as the practical **end of game theory**: the solution moves beyond the choice of to *cooperate or not*, focusing entirely on *designing an optimal split*.

Trust and Contract

While **cooperation** is the theoretical assumption that moves us beyond dilemmas, the negotiation over the split is, in reality, a **business negotiation**. In the real world, this assumed trust is often reinforced by a **Contract** to ensure both players honor and commit to the bargaining result.

GAME TRANSFORMATION

How to Discover the Outside Option?

Even though the Nash Bargaining game is considered the **end of game theory**, a problem remains: How do you know the other player's Outside Option?

To solve this, we can use the **Signaling Method** proposed in episode 4.1, the Lemon Market. By signaling your own Outside Option to the other player, you allow the bargaining process to flow toward a specific result. Without a signal, the players are bargaining in the dark.

Application

Real-World Example: Securing a First Job

Bargaining is a vital skill in the labor market, particularly during the transition into a first professional role. A common strategy involves finding a first job offer quickly to use as a **safety net**. Let this first job serve as your **Outside Option**.

Once this safety net is secured, a person can search for a second job with much more confidence. By signaling that they already have another offer, they increase their bargaining power with the second company. This moves them from a position of "needing" the job to "choosing" the job. However, for the bargaining to work, the requested salary must still match the person's skills. In this case, bargaining is not just about force, but about aligning your value with the resources you demand.

After the Game

After surviving storms of games, deadlines, and the wild chaos of university life, George and Leo finally stood at the finish line. One warm evening, they hosted a graduation party with their football teammates. Sara was there too, standing beside George, while Betty the cat pawed curiously at balloons, like she was inspecting the decorations for quality control.

Tom and his friends had somehow taken over the KTV machine, fighting loudly over the microphone, each insisting their song was next.

Amid the cheerful chaos, George and Leo slipped onto the balcony with two cold colas. For the first time in a while, they were quiet.

“Do you remember our first year?” George asked. “Back when we thought game theory was just… theory?”

Leo laughed. “And then we realized — it’s life. Every choice, every fight, every cooperation. From feeding Betty to picking Netflix.”

They clinked their bottles together, watching the city lights flicker below.

Although they were about to graduate, neither felt worried. By pure luck, or maybe strategic coordination, they both landed jobs in the same area.

That meant they would keep living together, sharing rent, adventures, and of course… responsibility for Betty, their big boss.

George glanced back inside, where Sara laughed with friends, and Betty batted at a ribbon tied to the new car keys. Leo smirked.

“Guess the game’s not over yet,” George said.

“Nope,” Leo replied. “It just leveled up.”

With that, the roommates, teammates, and lifelong friends stepped into their next stage, not as students of game theory, but as players of life, ready for whatever game came next.

FROM CONFLICT TO ALLY

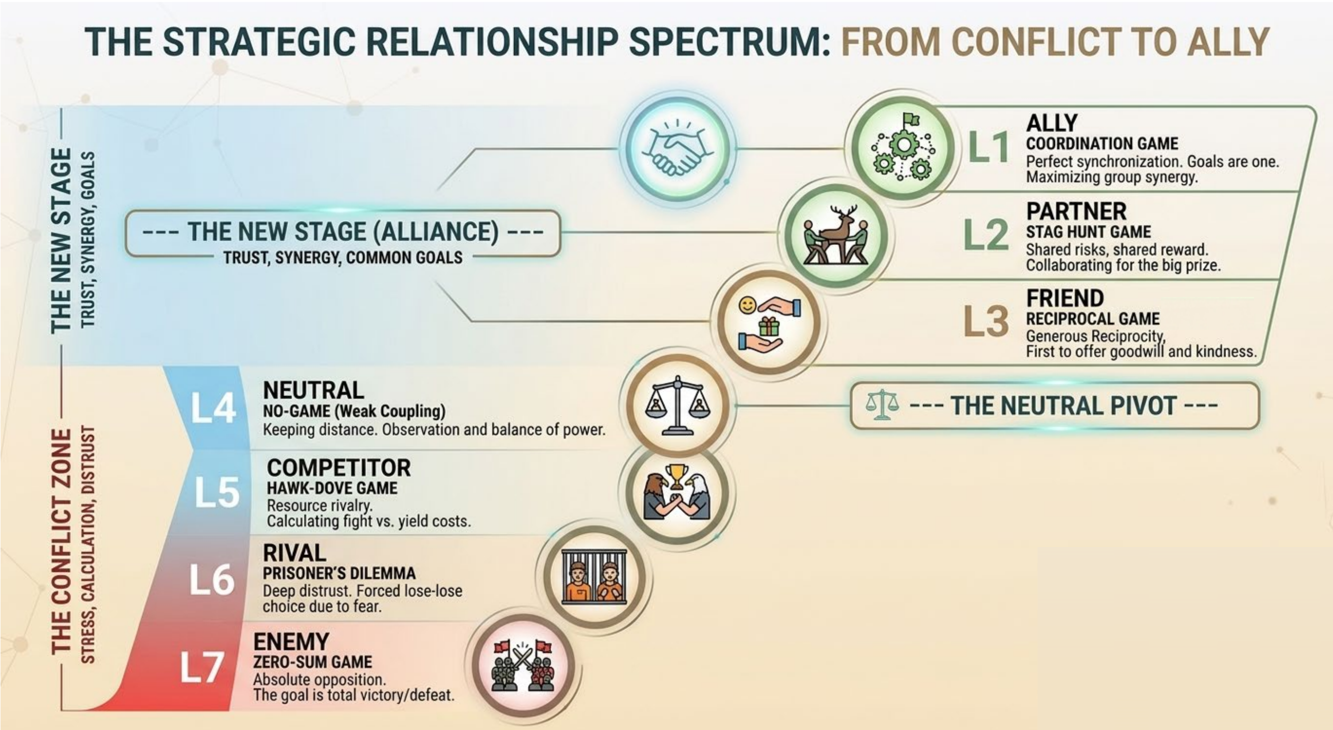


Figure C.1 The Strategic Relationship Spectrum

Within the framework of game theory, the distance between two players is not just a social feeling, but a mathematical reality. The spectrum from **Enemy** to **Ally** can be precisely divided into levels based on two variables: **the overlap of interests** and **the degree of adversarial intent**.

Level 7: The Enemy (Zero-Sum Game)

This is the lowest level of the relationship spectrum. Interests are in total conflict; the success or existence of one player is viewed as a direct threat to the other. This is a classic **Zero-Sum Game**, where one's gain is exactly equal to the other's loss. Often, it even devolves into a "Negative-Sum Game" where both sides suffer just to weaken the opponent. There are only hostile exclusion and all-out confrontation. In addition, there is no room for a pure strategy here.

In the Mixed strategy game episode, George & Leo wanted to avoid sitting next to Tom, while Tom's group wanted to follow them. No outcome could satisfy both groups simultaneously as one player's gain is another player's loss.

Level 6: The Rival (Prisoner's Dilemma)

A Rivalry represents strategic mistrust. It goes beyond only resource grabbing to include struggles over strategic dominance or market status. Here, players are trapped in the **Prisoner's Dilemma**. Although a better mutual choice exists, deep fear of the other's move prevents cooperation. The interaction is a constant cycle of high tension, Tit-for-tat exchanges, and strategic maneuvering, where players are forced into lose-lose choices.

During the Noise War, George and Leo end up with mutual chaos where both make it loud. George continues using a gaming keyboard while Leo keeps playing music on the speakers. Although they achieve a quiet space with cooperation.

Level 5: The Competitor (Hawk-Dove Game)

Competitors seek similar goals within a limited resource, such as a market or a reputation pool. Both sides play the **Hawk-Dove Game**, calculating whether the cost of fighting is worth the potential yield. Unlike Rivals, Competitors usually play within established rules or boundaries to avoid destructive conflict (Hawk, Hawk), using tactical jockeying to *test each other's limits* without burning the system down.

When Betty wakes up hungry at midnight, someone must feed her. There are two choices: go or not. If both go, they both lose sleep but live peacefully. If neither goes, they both suffer through Betty's meows.

Level 4: The Neutral Pivot (No-Game)

At the center of the spectrum sits the Neutral state, characterized by **No Strategic Interaction**. The goal paths of both parties do not overlap, meaning there is no incentive to cooperate but no direct reason for conflict. However, this is the most unstable state; it acts as a pivot point. Any environmental change, such as a sudden resource scarcity or a shift in technology, can instantly tilt a Neutral player toward becoming a Competitor or an Ally.

> Let's continue the Betty's Night Meal scenario as an example. What will happen during the neutral state is that George and Leo are pretending to sleep to test each other's limits while quietly calculating their own payoff between two choices. In this intense, unstable state, if Betty changes from meowing to knocking her bowl loudly. Leo and George might be melted, and both go feed her (becoming Ally), or they might get angry and decide to endure until someone gives up (becoming Competitor), as now no one wants to get close to the noisy cat.

Level 3: The Friend (Reciprocal Game)

A Friendship in game theory is built on **Generous Reciprocity**. Even if interests are not perfectly aligned, players engage in a reciprocal strategy where the first move is always an offer of goodwill. The game remains stable because both sides understand that helping the other today ensures they will be helped tomorrow. It is the first step toward high-level coordination, moving the relationship away from calculation and toward kindness.

> The best example explaining this reciprocal game is in the Dynamic Hawk-Dove game episode, where George and Leo fight for the last bun. However, George decided to offer the bun to Leo as he knew they are playing repeated reciprocal game. Helping Leo today would be beneficial for him in the future.

Level 2: The Partner (Hunter or Stag Hunt Game)

Partners share common interests in specific projects or phased goals, they are not fully integrated. This relationship is best described by the **Stag Hunt Game**. They are united by the realization that the largest rewards (the Stag) are only possible through corporation, sharing risk and labor. However, their relationship is based on specific agreements that have a clear division of work. They still maintain their own reserved domains; trust is conditional. Also, small-scale competition may exist in non-core areas.

> Consider a situation where Betty doesn't feel well and needs to go to the vet. This scenario involves a trade-off between securing the best care for Betty (requires cooperation, one to drive while the other holds her to keep Betty calm, high reward), and completing their individual homework that is due soon (does not require cooperation, forget Betty for now, safe solo reward). They will cooperate only if they can agree on the division of labor (who drives, who holds Betty, who pays the bills.)

Level 1: The Ally (Coordination Game)

This represents the highest peak of cooperation. An Alliance exists when interests are fully unified and long-term goals are identical. In this **Coordination Game**, the challenge is about synchronization (resource sharing, and collective defense). Because trust is absolute and information is transparent, the players maximize group synergy. They are willing to make short-term sacrifices for one another in exchange for a long-term Positive-Sum Game (Win-Win), where they rise and fall as one.

> In the Netflix or Disney+ scenario, George and Leo would like to share streaming subscriptions to save money. If they both pick the same platform, they save money and can watch together. But if they choose different ones, no shared account, and double the cost. After communication, they build trust easily as they have a unified goal.

A Note on the Perfect Cooperation: When players fully trust each other, the strategic *game* essentially disappears. This spectrum focuses on the stages where players are still engaged in a game due to **conditional trust**. In these cases, cooperation only becomes stable when it is guaranteed by a **Contract** or a **Third-party** enforcer, which removes the risk of betrayal.

FROM MARKET FAILURE TO FULL COOPERATION

Six Market Interaction Models Spectrum
(From Market Failure to Full Cooperation)

Type	Market State	Game Theory Model	Nature of Interaction
M1	Perfect Collaboration	Cooperative Game	**Mutual benefit and fair distribution:** Parties share information and resources, achieving win-win outcomes through mutual trust and collaboration.
M2	Institutionalized Competition	Auction Mechanism	**Rule-based and resource allocation:** Through a public and competitive auction process, the price and allocation of resources are determined.
M3	Peer-to-Peer Bargaining	Bargaining Model	**Price negotiation and mutual concessions:** Parties negotiate and seek mutually acceptable outcomes.
M4	Integrity Risk	Moral Hazard	**Information asymmetry and behavioral risk:** After transactions, information becomes asymmetric, leading to potential opportunistic behavior.
M5	Market Breakdown	Adverse Selection	**Poor quality drives out good quality:** Information asymmetry before transactions leads to the elimination of high-quality options.
M6	Trust Failure	Prisoner's Dilemma	**Loss of trust and failure of cooperation:** Lack of trust and inability to commit, making cooperative transactions impossible.

Figure C.2 Market Interaction Models Spectrum

This spectrum illustrates the different ways markets interact, moving from ideal collaboration down to complete breakdown. It serves as a map for understanding how trust and information determine the success or failure of any human exchange.

The Failure Zone (M6)

Deals do not happen at all because the lack of trust makes cooperation impossible. This is a total collapse where no one dares to make a move.

M6: Trust Failure (Prisoner's Dilemma)

This is the absolute bottom of the spectrum. Even if a deal would benefit everyone, the total lack of trust prevents anyone from committing. Both sides choose a lose-lose path, because they are too afraid of being exploited. This is the same logic behind destructive price wars or global arms races.

> Recall the Noise War, where George and Leo were stuck in a cycle of blasting noise to annoy each other. Neither George nor Leo would reduce their noise first, because they didn't trust the other to do the same. If this situation continues, both players eventually realize that cooperation is impossible. They will end their mutual housing contract and move out. The entire structure of their relationship collapses.

The Distortion Zone (M4-M5)

Deals are made, but they are weakened by hidden information or unreliable behavior like hidden actions.

M5: Market Breakdown (Adverse Selection)

Problems arise **before** the deal due to hidden information. When buyers cannot distinguish between high-quality and low-quality options, they become unwilling to pay a premium price. As a result, high-quality participants exit the market, leaving only the bad ones behind.

> The best example is **Lemon Market** in used cars, where the sellers know the true quality of their vehicles, but buyers, including George, does not. If George sees many bad reviews, he loses confidence. To protect himself, he is only willing to pay

a safe, low price…just in case the car is a lemon. Sellers of high-quality will then leave the market, because they know their cars are worth more than George's low offer. Now, only the sellers of lemons remain. The market fails, leaving buyers with nothing but bad options.

M4: Integrity Risk (Moral Hazard)

This is the first stage of market trouble. A deal is signed, but because one party's actions are hidden **after** the agreement, they may act opportunistically or take "behavioral risks." For example, employees slacking off after getting a permanent contract, or insurance fraud.

Consider a scenario where the Professor announces a new grading policy (The bottom 10% of the class get a C, and the class average will be a B), as he assumes students will learn better in a relaxed environment. However, after knowing this, Leo's behavior changes. He hiddenly reduces his effort. Since his poor performance is now protected by the safety net, the Professor cannot easily recognize if Leo's lower grades are due to the difficulty of the material or a simple lack of effort.

The Efficiency Zone (M1-M3)

In this zone, deals happen successfully through trust, cooperation, established rules, or direct negotiation.

M3: Peer-to-Peer Bargaining (Bargaining Model)

When there is no middleman or auction system, parties must negotiate directly. This involves a cycle of offers and counter-offers to find a mutually acceptable compromise. This model is commonly seen in salary negotiations or marketplace haggling.

As seen in the **Outside Option** episode, if George and Leo decide to order food together to get a discount, they must bargain over how to split the savings.

George: "Hey man, since we're both hungry, why don't we just pair up? I'll give you $10 of the discount. Deal?"
Leo: "Nice try, bro. But Tom already offered me $45. That's my floor. I'm not taking anything less than $80 from you if we do this."
George: "Eighty? No way. Sara is already giving me $30 for zero effort. Let's be fair~"

The negotiation moves toward a Nash Equilibrium. They will first pay out their respective outside options ($30 for George and $45 for Leo), then split the remaining mutual surplus evenly. This leads to a final split of $42.50 for George and $57.50 for Leo.

M2: Institutionalized Competition (Auction Mechanism)

Here, trust between individuals is not necessary because the **system** enforces the rules. Allocation and pricing are determined through transparent processes like auctions. Real-world examples of the M2 type include stock markets or government spectrum auctions.

Consider Leo participating in an **auction** for a poster, signed by his favorite singer on a platform like Facebuck Marketplace. Whether the format is a descending, ascending, second-price or first-price auction, Leo does not need to know or trust the other bidders. He enters the game because he believes in the fair winning rules and the transparent bidding process.

M1: Perfect Collaboration (Cooperative Game)

In the best-case scenario, all parties share information and resources transparently. Because there is deep mutual trust, the focus shifts to maximizing group synergy and ensuring a fair, Win-Win distribution. In reality, these perfect collaborations are seen in successful joint ventures or specialized research partnerships.

The perfect example for this is a **Symmetric Coordination Game.** Consider the scenario of George and Leo choosing between Netflix or Disney+. Both have a strong incentive to pick the same platform, so they can share the experience and the cost. This shared incentive builds deep trust, and when they are transparent about their preferences, they easily coordinate on the same choice.

In conclusion, the movement along this spectrum is driven by two critical forces:

1. **Information Symmetry:** Does everyone know the same facts?
2. **Trust & Enforceability:** Can the agreement be relied upon?

As these forces weaken, the market loses its ability to create value and slides downward from **Cooperation** toward **Failure**.