Chapter 9 Markets for Differentiated Product

- Market for Two Differentiated Products
- Monopolistic Competition in Differentiated Products
- Linear location model
- Circular location model

Market for Two Differentiated Products

- Dixit (1979), Singh and Vives (1984)
- We assume the demand functions for the two products as

$$p_1 = \alpha - \beta q_1 - \gamma q_2 \qquad where\ \beta > 0,\ \beta^2 > \gamma^2$$
$$p_2 = \alpha - \gamma q_1 - \beta q_2$$

The smaller γ the higher product differentiation

- Inverse demand functions

$$q_1 = a - bp_1 + cp_2 \qquad where\ a = \frac{\alpha(\beta-\gamma)}{\beta^2-\gamma^2}, b = \frac{\beta}{\beta^2-\gamma^2} > 0, c = \frac{\gamma}{\beta^2-\gamma^2}$$
$$q_2 = a + cp_1 - bp_2$$

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Market for Two Differentiated Products (cont')

- Quantity game (Cournot game) with differentiated products

$$\max_{q_i} \pi_i = (\alpha - \beta q_i - \gamma q_j) q_i \quad i, j = 1,2, i \neq j$$

- Best response functions given by

$$q_i = R_i(q_j) = \frac{\alpha - \gamma q_j}{2\beta} \quad i, j = 1,2, i \neq j$$

- Symmetric equilibrium

$$q_i^c = \frac{\alpha}{2\beta+\gamma}, p_i^c = \frac{\alpha\beta}{2\beta+\gamma}, \pi_i^c = \frac{\alpha^2\beta}{(2\beta+\gamma)^2}, i = 1,2$$

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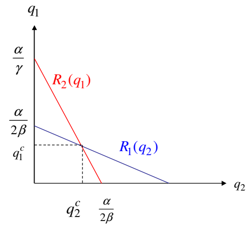
Market for Two Differentiated Products (cont')

- Quantity game (Cournot game) with differentiated products



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Market for Two Differentiated Products (cont')

- Price game (Bertrand game) with differentiated products

$$\max_{q_i} \pi_i = (a - bp_i + cp_j) p_i \quad i, j = 1,2, i \neq j$$

- Best response functions given by

$$p_i = R_i(p_j) = \frac{a + cp_j}{2b} \quad i, j = 1,2, i \neq j$$

- Symmetric equilibrium

$$p_i^c = \frac{a}{2b-c} = \frac{\alpha(\beta-\gamma)}{2\beta-\gamma}, p_i^b = \frac{ab}{2b-c}, \pi_i^b = \frac{a^2 b}{(2b-c)^2} = \frac{\alpha^2\beta(\beta-\gamma)}{(\beta-\gamma)^2(\beta+\gamma)}$$

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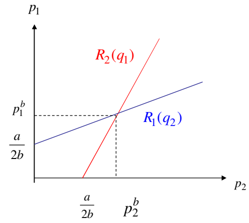
Market for Two Differentiated Products (cont')

- Price game (Bertrand game) with differentiated products



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Market for Two Differentiated Products (cont')

- Cournot versus Bertrand in differentiated products

$$p_i^c - p_i^b = \frac{\alpha}{4\left(\beta^2/\gamma^2\right)-1}$$

$$\Rightarrow \quad p_i^c - p_i^b > 0, \frac{\partial(p_i^c - p_i^b)}{\partial\gamma} > 0, \lim_{\gamma\to 0}[p_i^c - p_i^b] = 0$$

The smaller γ the higher product differentiation

Monopolistic Competition in Differentiated Products

- Monopolistic-competition environment (Chamberlin 19330)
- The environment
  - (1)Consumers are homogeneous and can be represented by a single consumer who loves to consume a variety of brands
  - (2) There is an unlimited number of potentially produced brands
  - (3) Free entry of new-producing firms

Monopolistic Competition in Differentiated Products (cont')

- The utility of the representative consumer is given by a constant-elasticity-of-substitution (CES) utility

$$u(q_1,q_2,\ldots,q_N) = \sum_{i=1}^{N}\sqrt{q_i} \quad \Rightarrow \quad \eta = \frac{\partial q_i}{\partial p_i}\cdot\frac{p_i}{q_i} = -2$$

- Each brand is produced by a single firm. The total cost of a firm producing qi unit of brand I is given by

$$TC_i(q_i) = \begin{cases} F + cq_i & if\ q_i > 0 \\ 0 & if\ q_i = 0 \end{cases}$$

Monopolistic Competition in Differentiated Products (cont')

- Resource constraint: Labor demanded for production equals the total labor supply

$$\sum_{i=1}^{N}(F + cq_i) = L$$

- Solving for a monopolistic-competition equilibrium

$$MR_i(q_i) = p_i(1+\frac{1}{\eta}) = \frac{p_i}{2} = c = MC(q_i) \quad \Rightarrow \quad p_i = 2c$$

$$\pi_i(q_i^{mc}) = (p_i^{mc} - c)q_i^{mc} - F = cq_i^{mc} - F = 0 \quad \Rightarrow \quad q_i^{mc} = \frac{F}{c}$$

$$N(F + c(F/c)) = L \quad \Rightarrow \quad N = \frac{L}{2F}$$

Monopolistic Competition in Differentiated Products (cont')

- Monopolistic competition in international markets
  - When the world is integrated into a single large economy, the labor resource and the number of consumers basically doubles
- Autarky market vs free trade market

$$p_i^f = 2c = p_i^a \quad q_i^f = \frac{F}{c} = q_i^a \quad N^f = \frac{L}{F} > \frac{L}{2F} = N^a$$

$$\Rightarrow \quad u^f = N^f\sqrt{q_i^f} = \frac{L}{\sqrt{2}\sqrt{cF}} > \frac{L}{\sqrt{2cF}} = N^a\sqrt{q_i^a}$$

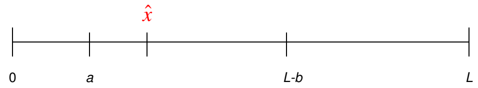
The linear "location " model

- Hotelling (1929) considers consumers who reside on a linear street with a length of *L*>0
- Suppose that the consumers are uniformly distributed on the interval, so that at each point lies a single consumer
- The total number of consumers in the economy is *L*
- Each consumer is indexed by $x \in [0, L]$, so *x* is just a name of consumer (located at point x from the origin)

The linear "location " model (cont')

- Suppose that there are two firms selling a product that is identical in all respects except one characteristic, which is the location where is sold
  - Firm *A* is located *a* units of distance from point 0. Firm B is located to the right of firm A, *b* units of distance from point *L*
- Each consumer buys one unit of product
  - A consumer has to pay transportation cost o τper unit of distance

The linear "location " model (cont')



Define the utility function of a consumer located at point x by

$$U_x = \begin{cases} -p_A - \tau \mid x - a \mid & \text{if he buys from A} \\ -p_B - \tau \mid x - (L-b) \mid & \text{if she buys from B} \end{cases}$$

Indifferent customer $\hat{x}$ is given by

$$-p_A - \tau(\hat{x} - a) = p_B - \tau(L - B - x)$$

$$\Rightarrow \quad \hat{x} = \frac{p_A - p_B}{2\tau} + \frac{L - b + a}{2}$$

The linear "location " model (cont')

The demand functions are

$$q_A = \frac{p_A - p_B}{2\tau} + \frac{L-b+a}{2} \qquad q_B = L - \hat{x} = \frac{p_A - p_B}{2\tau} + \frac{L-b+a}{2}$$

If $a < \hat{x} < L - b$ then

Indifferent customer $\hat{x}$ is given by

$$-p_A - \tau(\hat{x} - a) = p_B - \tau(L - B - x)$$

$$\Rightarrow \quad \hat{x} = \frac{p_A - p_B}{2\tau} + \frac{L - b + a}{2}$$

The linear "location " model (cont')

The profit functions are

$$\max_{p_A} \pi_A = \frac{p_B p_A - (p_A)^2}{2\tau} + \frac{(L-b+a)p_A}{2} \qquad \max_{p_B} \pi_B = \frac{p_B p_A - (p_B)^2}{2\tau} + \frac{(L-b+a)p_B}{2}$$

Solve FOC simultaneously

$$\frac{\partial \pi_A}{\partial p_A} = \frac{p_B - 2p_A}{2\tau} + \frac{(L-b+a)}{2} = 0 \qquad \frac{\partial \pi_B}{\partial p_B} = \frac{p_A - 2p_B}{2\tau} + \frac{L-b+a}{2} = 0$$

Bertrand-Nash equilibrium

$$\Rightarrow \quad p_A{}^h = \frac{\tau(3L-b+a)}{3} \qquad p_B{}^h = \frac{\tau(3L-b+a)}{3}$$

The linear "location " model (cont')

The equilibrium demand and profit are

$$\hat{x}^h = \frac{3L-b+a}{6} \qquad \pi_A{}^h = p_A{}^h \cdot \hat{x}^h = \frac{\tau(3L-b+a)^2}{18}$$

⇨ If *a*=*b*, then the market is equally divided between the two firms

Proposition
(1) If both firms are located at the same point (a+*b*=*L*), then $p_A=p_B=0$ is a unique equilibrium
(2) A unique equilibrium exists if and only if the two firms are not too close ti each other

$$\left(L + \frac{a-b}{3}\right)^2 \geq \frac{4L(A+2b)}{3} \text{ and } \left(L + \frac{b-a}{3}\right)^2 \geq \frac{4L(A+2b)}{3}$$

The linear "location " model (cont')

Location and price game

$$\frac{\partial \pi_A{}^h}{\partial a} > 0$$

⇨ Firms tend to move toward the center (minimum differantiation)
No equilibrium exist !

If the transport cost is quadratic

$$U_x = \begin{cases} -p_A - \tau(x-a)^2 & \text{if he buys from A} \\ -p_B - \tau[x-(L-b)]^2 & \text{if she buys from B} \end{cases} \quad \Rightarrow \quad \frac{\partial \pi_A{}^h}{\partial a} < 0$$

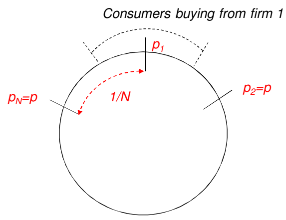
⇨ Firm A would choose *a*=0, and firm B would locate at point *L*

The circular location model

- Consumers are uniformly distributed on the unit circle. We denote by τ the consumers' transportation cost per uni of distance
- Assuming that N firms are located at an equal distance from one another yields that the distance between any two firms is $1/N$
- Assuming that firm 2 and N charge a uniform p, the consumer who is indifferent to whether buys firm 1 or firm 2 is located at $\hat{x}$, which is determined by $p_1 + \tau\hat{x} = p + \tau(1/N - x)$

$$\Rightarrow \hat{x} = \frac{p - p_1}{2\tau} + \frac{1}{2N}$$

The circular location model (cont')



The circular location model (cont')

Demand function facing firm *i*

$$q_i = 2\hat{x} = \frac{p - p_i}{\tau} + \frac{1}{N}$$

Profit function facing firm *i*

$$\pi_i = p_i q_i - (F + c q_i) = (p_i - c)\left(\frac{p - p_i}{\tau}\right) - F$$

Solve FOC

$$\frac{\partial \pi_i}{\partial p_i} = \frac{p - 2p_i + c}{\tau} + \frac{1}{N} = 0$$

Symmetric equilibrium

$$p_i{*} = p{*} = \frac{p{*} - 2p_i + c}{\tau} + \frac{1}{N}$$

The circular location model (cont')

Assume a free entry market, therefore equilibrium number of brands N is given by

$$\pi_i = \frac{\tau}{N^2} - F = 0 \quad \Rightarrow \quad N_m{}^{*} = \sqrt{\frac{\tau}{f}}$$

Social optimal number is given by solving

$$\min_N L = NF + T(N) = NF + 2N\tau \int_0^{\frac{1}{2N}} x\,dx = NF + \frac{\tau}{4N}$$

$$\Rightarrow \quad N_w{*} = \frac{1}{2}\sqrt{\frac{\tau}{F}} < N_m{}^{*}$$