Chapter 7 Incentive Mechanism

- Principle-Agent Problem
- Production with Teams
- Competition and Managerial Compensation
- Salary of Executive
- Regulating a Firm with Unknown Cost

Economic Incentives

- Motivation
  - Firms are organizations then are run and operated by people who use the technology to manufacture the products, and then set quantity and price to maximize profits in a given market structure
  - Firms are not own by their employees. What motives workers and managers to devote efforts leading to increasing the firm's profitability
  - How can the firm reward its workers and managers if the relationship between an individual and output cannot be observe
- Goal
  - Develop economic mechanisms that would provide the workers with (monetary) incentives to exert effort in their work

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Principal-Agent Problem

- The problem
  - Exists in almost every social structure where some units are regarded as managers and some units as supervised agents
  - Efforts are not observable but outcome can be observed
- Example
  - Parent vs kids (outcome: grade)
  - Landlord vs tenant (outcome :crops)
  - Plaintiff vs attorney (outcome: win or loss)
  - School vs professor (outcome: students' achievement)
  - Government vs worker (outcome: public polls)

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Economic Incentives under Certainty

- The agent (e.g. The waiter)
  - Denote e the amount of effort put by the agent
  - If the agent works hard, he puts effort level given by $e=2$
  - If he shirks, he puts an effort level given by $e=0$
  - Assume the agent's reservation utility $U=10$

    $U=w-e$ if he devotes an effort level e

    $=10$ if he works at another place
- The principle (e.g. The restaurant)
  - The revenue of the restaurant depends on the waiter's effort and is denoted by $R(e)$

    $R(e)=H$ if $e=2$

    $=L$ if $e=0$
  - The restaurant's profit is denoted by π

    $\pi=R(e)-w$

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Economic Incentives under Certainty (cont')

- The contract
  - Denote w^H denote the wage rate that the principal promises to pay the agent when the revenue is H, and let w^L be wage paid to agent when the revenue *is* L
  - The agent's **participation constraints** is

    $w^H-2>=10$
  - The agent's **incentive constraints** is

    $w^H-2>=w^L - 0$

⇨ $w^H=12$; $w^L=10$

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Economic Incentives under Uncertainty

- The nature determines the value of $R(e)$, but our agent can affect the probability of each realization of $R(e)$ by choosing his effort level

  $R(2)=H$ probability 0.8

  $=L$ probability 0.2

  $R(0)=H$ probability 0.4

  $=L$ probability 0.6

  The participation constraint

  $0.8w^H+0.2w^L-2>=10$

  The incentive constraint

  $0.8w^H+0.2w^L-2>= 0.4w^H+0.6w^L-0$

⇨ $w^H=13$; $w^L=8$ ⇨ The expected wage will equals the waiter's reservation utility plus his effort level

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Principal-agent problem under asymmetric information

The owner's belief

$R(2)=H$ probability 0.8 $R(0)=H$ probability 0.4
$=L$ probability 0.2 $=L$ probability 0.6

The worker's belief (more risk averse)

$R(2)=H$ probability 0.7 $R(0)=H$ probability 0.4
$=L$ probability 0.3 $=L$ probability 0.6

The participation constraint becomes

$0.7w^H+0.3w^L-2>=10$

The incentive constraint becomes

$0.7w^H+0.3w^L-2>=0.4w^H+0.6w^L-0$

⇨ $w^H=14$; $w^L=22/3$

⇨ The expected wage will exceeds the waiter's reservation utility plus his effort level

Production with Teams

- Problem
  - Inability to monitor a worker's effort generates an inefficiency
  - The output of the team depends on the effort levels of all workers assigned to work on a certain project
  - The group as a whole is rewarded on the value of the project, that is, when the individual workers are not rewarded according to their individual effort level
- Free-rider effect
  - a worker, knowing that all other workers in a team are putting a lot of effort into the project, will have an incentive to shirk

Production with Teams (cont')

- Consider a research lab developing the future product whose value is denoted by V
- There are N scientists who work on the project
- We denote by e_i the effort put in by scientist, i=1,2,…,N
- The value of the jointly developed product depends on the effort levels of all the N scientists and is given by

$$V=\sum_{i=1}^{N}\sqrt{e_i}$$

- We denote by w_i the compensation given to scientist i after the project is completed $\sum_i w_i = V$
- All scientists have identical preferences, summarized by the utility function $U_i = w_i - e_i$

A digression: Optimal effort levels

- Suppose that each scientist can observe the efforts of his other colleges, and they collude maximize their utility levels
- We set $e_i=e$ and $w_i=w=V/N$
- The representative effort e* that maximizes a representative worker's utility solves

$$\max(w-e)=\frac{V}{N}-e=\frac{N\sqrt{e}}{N}-e$$

⇨ $e^*=1/4 \quad V^*=N/2$

The equal-division economic mechanism

- Nash equilibrium in effort level
  - Each scientist takes the effort levels of his colleagues as given and chooses his effort level to maximize his utility

$$\max_{e_i} U_i=\frac{\sum_{j\neq i}\sqrt{e_j}+\sqrt{e_i}}{N}-e_i$$

FOC ⇨ $e^n=e_i=\frac{1}{4N^2}\le e^*$

⇨ $V^n=N\sqrt{e^n}=\frac{1}{2};$

Equilibrium utility $U_i=\frac{V^n}{N}-e^n=\frac{1}{2N}-\frac{1}{4N^2}$ ⇨ $\frac{\partial U_i}{\partial N}<0$

An economic mechanism that works

- If the team as a group achieves the optimal output V^*, then each team member receives V^*/N. If the teams output is different from V^*, then all team members receive 0. Formally

$$w_i=\begin{cases}V^*/N & if \sum_{i=1}^{N}\sqrt{e^*}=V^* \\ 0 & otherwise\end{cases}$$

Problem: time inconsistency; Worker anticipate that the manager will renegotiate the contract

Competition and Managerial Compensation

- Fersgtman and Judd (1987)
- Managerial compensation under duopoly
- Consider a market for a single homogeneous product, where the demand curve is given by
  $p=a-Q$
- The profit of each firm i, i=1,2 are given by
  $R_i=pq_i=(a-q_1-q_2)q_i$ and $\pi_i=R_i-cq_i$
- Assume the manager of each firm i is promised payment of a faction μ_i of linear combination of the firm's profit and the revenue
- Assume α_i is the faction of linear combination, $\alpha_i=1$ means the manager maximizes firm i's profit. $\alpha_i=0$ means the manager maximizes firm i's revenue
- Incentive to managers
  - Let M_i denote the compensation to manager of firm i
    $M_i=\mu_i[\alpha_i\pi_i+(1-\alpha_i)R_i]=\mu_i[(a-q_1-q_2)q_i-\alpha_i cq_i]$

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Competition and Managerial Compensation (cont')

- Two stage-decision-level market game
  - In the first stage the owner of each firm i choose μ_i and α_i to maximize the owner's profit given by
    $\pi(\mu_i,\alpha_i)=\pi_i-M_i$
  - In the second stage, managers choose output levels

Solve $q_i=\frac{a-q_j}{2}-\frac{\alpha_i c}{2}, i=1,2$

⇨ $q_i=\frac{a+\alpha_j c-2\alpha_i c}{3}, i=1,2$ $\quad Q=\frac{2a-\alpha_1 c-\alpha_2 c}{3}$

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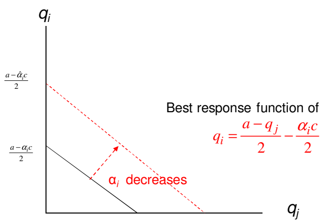
Competition and Managerial Compensation (cont')



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Competition and Managerial Compensation (cont')

- Demand function becomes
  $$p=a-Q=\frac{a+\alpha_1 c+\alpha_2 c}{3}$$
- Firm i 's decision
  $$\max_{\alpha_i}\pi_i=[p-c]q_i=\frac{[a+c(\alpha_i+\alpha_j-3)][a+c(\alpha_j-2\alpha_i)]}{9}$$

FOC ⇨ $\alpha_i=\frac{6c-a-c\alpha_j}{4c}$

Symmetric Nash equilibrium

$$\alpha^e=\alpha_1{}^e=\alpha_2{}^e=\frac{6c-a}{5c}$$

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Competition and Managerial Compensation (cont')

If α_2=1 (profit maximizing), then $\alpha_1=\frac{5c-a}{4c}$

$q_1=\frac{a+\alpha_2 c-2\alpha_1 c}{3}=\frac{a-c}{2}$ ⇨ Same result as leader and follower game

$q_2=\frac{a+\alpha_1 c-2\alpha_2 c}{3}=\frac{a-c}{4}$

⇨ By employing managers, firm 1 can advance the strategic position of his firm beyond what could be achieved if the owner was managing the firm by himself

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Competition and Managerial Compensation (cont')

⇨ $q_i{}^e=\frac{a-\alpha^e c}{3c}=\frac{2(a-c)}{5}>q_i^c=\frac{a-c}{3}$

If collusion between the owner

Firms' decision problem becomes

$$\max_{\alpha}(\pi_1+\pi_2)=2(a\alpha c-2\alpha^2c^2+3\alpha c^2)$$

$$\alpha^*=(a+3c)/(4c)$$

⇨ $q_i{}^*=\frac{(a-c)}{4}<q_i^c=\frac{a-c}{3}$

Collusion among owners yields lower output levels and higher profit to each firm than under the Cournot competition

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Why Executives Are Paid More than Worker

- Assume there are only two workers. One of them will be promoted and will become a manager/executive
- The promotion is granted to the worker who will turn over a higher output level

Why Executives Are Paid More than Worker (cont')

The relationship between the effort level of employee *i* and his or her effort

$$q_i = \begin{cases} 0 & \text{If } e_i=0 \\ H \quad \text{probability } 1/2 & \text{If } e_i=e \\ 0 \quad \text{probability } 1/2 & \end{cases}$$

The probability worker 1 will be promoted to a rank of executive

$$p = \begin{cases} 1/2 & if\ e_1 = e_2 = e \\ 1/2 & if\ e_1 = e_2 = 0 \\ 3/4 & if\ e_1 = e \text{ and } e_2 = 0 \\ 1/4 & if\ e_1 = 0 \text{ and } e_2 = e \end{cases}$$

Why Executives Are Paid More than Worker (cont')

Condition for both worker exert high effort *e*

$$EU_1(e,e) = 0.5w^E + 0.5w^W - e > 0.25w^E + 0.75w^W = EU_1(0,e)$$

$$EU_1(e,0) = 0.75w^E + 0.25w^W - e > 0.5w^E + 0.5w^E = EU_1(0,0)$$

$$\Rightarrow \quad w^E > 4e + w^W$$

Regulating a Firm under Unknown Cost

- State government agencies are assigned to determine the price that public utility companies charge their customers and provide lump-sum subsidy to cover the fixed cost
- Unit cost *c* is known to the company but unknown to the government
- *c* could be c^H with a probability of ρ and c^L with a probability of 1- ρ
- The government determine the price $p(\hat{c})$ and subsidy $s(\hat{c})$ according to the revelation of the company cost $\hat{c}$

Regulating a Firm under Unknown Cost (cont')

- Truthful revelation and the profit of the regulated firm
  - We denote by $\pi(\hat{c},c)$ the profit of the firm with a true unit production cost *c* that reports to have a unit cost of $\hat{c}$ to the regulating agency
  - For every value *c* and $\hat{c}$, the firm's profit is given by

    $$\pi(\hat{c},c) = p(\hat{c})Q - cQ + s(\hat{c}) = [p(\hat{c}) - c][a - p(\hat{c})] + s(\hat{c})$$

  - The profit of the firm when it reveals it true cost parameter, that is when $c = \hat{c}$

    $$\pi^*(c) = \pi(c,c) = [p(c) - c][a - p(c)] + s(c)$$

Regulating a Firm under Unknown Cost (cont')

- Definition: An economic mechanism $p(\hat{c}), s(\hat{c})$ is said to be satisfy the property called
  - 1. incentive compatibility if the firm cannot increase its profit by not reporting its true parameter. That is , for every $\hat{c} \in \{c^H, c^L\}$

    $$\pi(\hat{c},c) \le \pi(c,c) = \pi^*(c)$$

  - 2. individual rationality if the firm makes a nonnegative profit when it is reporting its true cost parameter. That is, $\pi(c,c) = \pi^*(c) \ge 0$

Regulating a Firm under Unknown Cost (cont')

- $p(\hat{c}), s(\hat{c})$ satisfy
  - 1. incentive compatibility

$$[p(c^H)-c^H][a-p(c^H)]+s(c^H)\geq[p(c^L)-c^H][a-p(c^L)]+s(c^L)$$
$$[p(c^L)-c^L][a-p(c^L)]+s(c^L)\geq[p(c^H)-c^L][a-p(c^H)]+s(c^H)$$

  - 2. individual rationality

$$[p(c^H)-c^H][a-p(c^H)]+s(c^H)\geq 0$$

$$[p(c^L)-c^L][a-p(c^L)]+s(c^L)\geq 0$$

Regulating a Firm under Unknown Cost (cont')

- Economic mechanism $\begin{cases} p(c^H)=c^H, s(c^H)=c^H \\ P(c^L)=c^L, s(c^L)=c^L \end{cases}$ satisfy the following conditions can induce the firm to reveal its true cost

$$\begin{cases} (c^H-c^L)(a-c^H)\leq s(c^L)-s(c^H)\leq(c^H-c^L)(a-c^L) \\ s(c^H)\geq 0 \\ s(c^L)\geq 0 \end{cases}$$

⇨ $s(c^L)>s(c^H)$ Subsidy to (reported) low cost should be (reported) higher than high cost